Financing of Firms, Labor Reallocation and the Distributional Role of Monetary Policy

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Abstract

We analyze monetary policy in a heterogeneous firms environment where cash constrained firms finance operations through external financing and cash unconstrained firms operate by using internal funds. We show that firms respond differently to shocks: expansionary monetary policy sharply increases the relative employment of the cash constrained firms while positive productivity shocks induce a rise in the relative employment of the cash unconstrained firms. Our analysis points to a clear role of monetary policy in reallocating resources across sectors that differ in their financing capabilities. Furthermore, the predictions of our model match the empirical evidence implying that financially constrained firms react sharply to monetary policy shocks but are less cyclical than unconstrained firms following productivity shocks.

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1 Introduction

Firms are heterogenous in terms of financing opportunities in that a large share of them depends on borrowing in order to finance operations.¹ In addition, empirical evidence shows that heterogeneous firms react differently to economic shocks ([Gertler and Gilchrist, 1994](#); [Jansen and Tsai, 2010](#); [Kudlyak et al., 2010](#); [Moscarini and Postel-Vinay, 2012](#); [Chari et al., 2013](#); [Fort et al., 2013](#); [Gilchrist et al., 2014](#); [Kudlyak and Sanchez, 2016](#)). We present a quantitative model in order to understand the underlying sources behind the differences in the cyclical sensitivity of heterogenous businesses. Specifically, we construct a limited participation monetary model where one type of firms uses internal funds and the rest rely on costly external financing in order to operate. The model predicts that firms that finance production externally are less cyclical after productivity shocks compared to firms that use internal financing. On the contrary, we find that firms that depend on external financing are more responsive to monetary policy shocks relative to firms that finance production internally. The responses across firms vary both in terms of relative production and the relative amount of labor. Thus, we find that the usual economic shocks affect not only total labor and production in the economy but also the allocation of resources across firms. We also show that, relative to productivity shocks, monetary policy shocks have profounder distributional effects.

In our baseline model, the two types of firms differ only in their financing schemes: the cash constrained firms need to borrow in advance in order to operate while the cash unconstrained firms operate using internal sources. Therefore, changes in relative employment across the two sectors are driven by changes in the cost of external financing. Expansionary monetary policy decreases the cost of borrowing, which directly benefits the constrained firms and leads to a rise in their employment and production. Importantly, while a monetary policy expansion leads to a rise in employment and production of the unconstrained firms as well, the allocation of labor tilts in favor of the constrained ones. In this respect, monetary policy is not only effective in stimulating aggregate economic activity but also plays a great role in the distribution of economic activity across sectors. The role of monetary policy in allocating resources across sectors, however, may have undesirable consequences; if the constrained firms are less productive than the unconstrained firms, then a monetary policy expansion tilts resources towards the less productive firms, which renders monetary policy less effective in stimulating output.

Apropos of productivity shocks, our model suggests that a positive shock induces an in-

¹According to [Rajan and Zingales, 1998](#), more than 30% of the US firms' operation depends on external finance

crease in the production of both types of firms. Yet, the surge in economic activity increases the interest rate and thus benefits the unconstrained firms more than the constrained ones. The relative employment of the unconstrained firms rises, making the unconstrained firms more cyclical following productivity shocks. In the baseline model, where we assume the same productivity for all firms, the effects of productivity shocks on the allocation of labor across sectors is considerably smaller than the effects of monetary policy disturbances. When we allow higher productivity for the cash unconstrained firms or a muted rise in the productivity of the constrained firms (which could reflect a slower adoption of new production technologies by small firms), then the effects of productivity shocks on relative employment of the constrained and unconstrained firms are greatly magnified. Yet, the response of relative employment to productivity innovations remains weaker than its response to monetary policy shocks.

In order to relate our findings to the empirical literature on differential firms' behavior over the cycle and after shocks, we conjecture that small firms, in terms of assets' size or employment, but also young firms, are more likely to be cash constrained compared to large/old ones.² Then, we assume that it is the small/young firms that rely on external financing, and thus are subject to the external finance premium. This assumption finds ground in previous empirical work. For example, [Fazzari et al. \(1988\)](#) studying investment of US manufacturing firms, find that smaller firms are more likely to be liquidity constrained. [Gertler and Hubbard \(1988\)](#) find that smaller firms rely heavily on intermediary credit while large firms use more direct credit (equity or commercial paper). [Haltiwanger et al. \(2013\)](#) have highlighted the importance of firm age in understanding why firms react differently to shocks. [Adelino et al. \(2016\)](#) show that, compared to older firms, young firms are more likely to face more severe financing constraints, and that job creation by new firms is larger in areas with better access to small business financing.

The heterogenous responses of firms to economic shocks have been well documented in the empirical literature. [Gertler and Gilchrist \(1994\)](#) conclude that after [Romer and Romer \(1989\)](#) monetary policy shocks dates, small firms (i.e. firms with low gross nominal assets' value), react more than larger firms; they severely reduce their sales and inventories and, thus, are more cyclical than larger ones after monetary policy shocks. [Chari et al. \(2013\)](#) and [Kudlyak and Sanchez \(2016\)](#) verify these findings for longer time series. Assuming that

²While we do not explicitly model size or age of firms, there are few reasons behind our assumption that small/young firms are more sensitive to the external finance premium. For example, higher cash flows might make it easier for larger firms to finance production internally. Also, large/old firms might be traded so they can easier raise capital internally. These effects are supported by the discussion that follows on the empirical literature.

interest rate shocks in our model map onto the [Romer and Romer \(1989\)](#) monetary shocks dates, then the results that are generated by our model align with the empirical evidence on the effects of monetary policy disturbances on heterogenous firms.

Regarding productivity shocks, [Chari et al. \(2013\)](#) find that after NBER dates large firms react more than small firms, reducing more their sales and inventories. [Kudlyak et al. \(2010\)](#) and [Kudlyak and Sanchez \(2016\)](#) emphasize that this effect holds also for the recent Great Recession. [Moscarini and Postel-Vinay \(2012\)](#) find that large firms (in terms of employment), on net destroy proportionally more jobs relative to small firms during a typical recession, and that they create more jobs during a typical expansion. Our results align with the above empirical findings if we consider positive (negative) productivity shocks in our model mapping onto an average US expansions (recessions).

Comparing the relative importance of the two shocks in generating distributional effects, our model suggests that the distributional effects of productivity shocks are smaller than those of the monetary policy shocks. This occurs because productivity shocks in our model affect all firms, while monetary policy shocks affect directly only the constrained firms through the impact on the cost of borrowing. This finding is in line with the implications of [Chari et al. \(2013\)](#)' work who find that monetary policy shocks produce differences in the responses of large and small firms, namely small firms react considerably more than large firms; however, after NBER dates the differences across firms are smaller (i.e. large firms react more, but not considerably more, than small firms).

Previous literature on financial frictions and monetary policy has focused on the costly state verification of firms affected by idiosyncratic shocks ([Bernanke et al., 1999](#); [Carlstrom et al., 2010](#); [Fiore et al., 2011](#)). Our work concentrates on heterogenous firms in terms of their ability to use internal finance. This type of heterogeneity affects the way that aggregate productivity and monetary policy shocks transmit across different sectors of the model. In addition, it implies an important role for monetary policy in terms of the allocation of resources across the various sectors of the economy.

Another strand of the literature we relate to has studied the asymmetric effects that monetary policy has across heterogenous agents in the economy (e.g., [Alvarez et al., 2001](#); [Williamson, 2005](#); [Williamson, 2006](#); [Zervou, 2013](#)). Our model studies asymmetric effects of monetary policy too, focusing on firms' heterogeneity. We use a large family model ([Lucas, 1990](#)) and the limited participation literature ([Fuerst, 1992](#); [Christiano and Eichenbaum, 1995](#)), where firms are subject to a cash-in-advance constraint and borrow in order to pay upfront the inputs' suppliers. We enhance these models with firms' heterogeneity. Our work also derives intuition from [Palivos et al. \(1993\)](#), who use a modified cash-in-advance

model, where only a fraction of the firms' operation is financed with credit, in order to study velocity changes.

The rest of the paper is organized as follows. Section 2 introduces the model economy. Section 3 presents the parametrization of the model and Section 4 shows empirical evidence that supports our parameter choices. Section 5 presents our results and Section 6 performs robustness analysis. Section 7 introduces an alternative model specification by allowing for different productivity levels across firms. Section 8 concludes.

2 The Model Economy

We use a limited participation model that is inspired by Fuerst (1992) and Williamson (2005). We enrich these models with firms' heterogeneity, allowing a fraction of firms to be cash constrained, while the rest are unconstrained. Monetary policy is governed by a Taylor-type rule, and the economy is subject to monetary policy as well as productivity shocks.

2.1 Households

We employ a representation of the economy following Lucas (1990) where the model economy is populated by infinitely lived large families. Households derive utility from two consumption goods: $c_{c,t}$, which is produced by the constrained firms, and $c_{u,t}$, which is produced by the unconstrained firms. Households also supply labor H_t . All families are identical and seek to maximize their lifetime utility:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\mathcal{U}(c_{c,t}, c_{u,t}) - D(H_t) \right] \right\}, \quad (1)$$

with E_t being the expectations operator and $0 < \beta < 1$ the family's subjective discount factor. We assume that $\mathcal{U}_i \equiv \frac{\partial \mathcal{U}}{\partial c_{i,t}} > 0$, $\mathcal{U}_{i,i} \equiv \frac{\partial^2 \mathcal{U}}{\partial c_{i,t}^2} < 0$, $\mathcal{U}_i(0) = \infty$, $\mathcal{U}_i(\infty) = 0$, for $i = c, u$. Similarly, $D(\cdot)$ denotes disutility from working a fraction $H_t \in [0, 1]$ of time in period t , with $0 < D'(\cdot) < \infty$ and $D''(\cdot) > 0$.

We now describe how trade takes place in this model economy. Each household consists of three members: a shopper, a worker and a financial transactor. All family members leave the family at the same time and each of them executes a specific task. The shopper receives part of the family's initial money balances and travels to the goods markets to purchase consumption goods from the two industries. The financial transactor receives a

fraction of the initial money balances in order to make loans in the loanable funds market. Finally, the worker works for the two types of firms.

In order for money to be valuable, we assume a cash-in-advance constraint on the consumption of the two goods. Moreover, there is a cash-in-advance constraint in the production of the cash constrained firms. These firms, contrary to the cash unconstrained ones, cannot hire labor with credit, but rather need to have cash balances in advance in order to pay their wage bill in the beginning of the period.³

The cash constrained firms may borrow cash balances from the loanable funds market. The quantity of loanable funds supplied partly comes from the money balances that the financial transactor carries, N_t . The monetary authority alters the nominal interest rate through a Taylor-type rule, which in turn affects the cost of borrowing of these firms. In this way, monetary policy affects directly only the cash constrained firms, which are the ones that participate in the financial markets. Having the nominal interest rate being determined, the monetary authority transfers cash to the financial intermediary, T_t , so that $\bar{M}_t = \bar{M}_{t-1} + T_t$, where $\bar{M}_t$ is money supply in period t .

The shopper travels to the goods market where she buys consumption good $c_{c,t}$ from the cash constrained firms at the price $P_{c,t} \geq 0$, and consumption good $c_{u,t}$ from the cash unconstrained firms at the price $P_{u,t} \geq 0$. The worker travels to the labor market, where she may work for a cash constrained or unconstrained firm, earning in either case the competitive wage, $W_t \geq 0$. The financial transactor travels to the loanable funds market holding N_t amount of cash. Without loss of generality we will assume that for non-negative nominal interest rate paid on loans, $R_t \geq 0$, the intermediary lends out all its cash.

The profit that the intermediary attains, $T_t(1 + R_t)$, is transferred to the household at the end of the period.⁴ Consumption can only be financed with cash, according to the following cash-in-advance constraint:

$$M_{t-1} - N_t + W_t H_t \geq P_{c,t} c_{c,t} + P_{u,t} c_{u,t},$$

where M_{t-1} is the family's initial cash balances in period t .

The family's budget constraint is as follows:

$$M_{t-1} + W_t H_t + (1 + R_t) N_t + (1 + R_t) T_t + \Pi_t \geq M_t + N_t + P_{c,t} c_{c,t} + P_{u,t} c_{u,t},$$

³Our assumption of cash constrained and unconstrained firms has the flavor of purchasing a subset of goods with cash, as analyzed by [Lucas and Stokey \(1983\)](#).

⁴With the supply of loanable funds being $N_t + T_t = B_{s,t}$, the intermediary's profit is $N_t + T_t + B_{d,t}(1 + R_t) - N_t(1 + R_t) - B_{d,t}$. By imposing equilibrium on the funds market, i.e., equating demand and supply of loanable funds, $B_{d,t} = B_{s,t} = N_t + T_t$, profit is: $T_t(1 + R_t)$.

where Π_t is the combined profit of the two production industries. By dividing both sides of the cash-in-advance and budget constraint by the aggregate price level (P_t), the real version of those constraints reads:

$$\frac{m_{t-1}}{1 + \pi_t} - n_t + w_t H_t \geq p_{c,t} c_{c,t} + p_{u,t} c_{u,t}, \quad (2)$$

and

$$\frac{m_{t-1}}{1 + \pi_t} + w_t H_t + (1 + R_t) n_t + (1 + R_t) \tau_t + \widetilde{\Pi}_t \geq m_t + n_t + p_{c,t} c_{c,t} + p_{u,t} c_{u,t}, \quad (3)$$

with $\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}}$ being the inflation rate, $\widetilde{\Pi}_t$ the real profits, $p_{c,t}$ the relative price of the good produced by the constrained firms and $p_{u,t}$ the relative price of the good produced by the unconstrained firms. The rest of the lower-case variables correspond to the real value of their upper-case counterparts.

Optimization by households is described in Appendix A and yields:

$$\frac{\mathcal{U}_{c,t}}{\mathcal{U}_{u,t}} = \frac{p_{c,t}}{p_{u,t}}, \quad (4)$$

with $\mathcal{U}_{c,t}$ and $\mathcal{U}_{u,t}$ being the marginal utility from the good produced by the constrained firms and unconstrained firms, respectively. As expected, it suggests that the marginal rate of substitution (MRS) between the two types of good is equal to their relative price.

2.2 The Production Sector

Firms hire labor in a competitive labor market, produce and ship their products to the goods markets. There is a unit mass of constrained firms and similarly, a unit mass of unconstrained ones. Firms use the production functions $y_{u,t} = f^u(H_{u,t}, \theta_t) = \theta_t H_{u,t}$ and $y_{c,t} = f^c(H_{c,t}, \theta_t) = \theta_t H_{c,t}$, with $H_{c,t}$ and $H_{u,t}$ denoting labor of the constrained and unconstrained firms respectively. Note that we let the two types of firms differ only in their financing possibilities; therefore, we let total factor productivity $0 < \theta_t < \infty$ be common to all firms in both sectors.⁵

The cash unconstrained firms can operate through credit. After realizing the productivity and monetary shocks they decide how much labor to hire. They promise to pay the worker the competitive wage w_t , which they pay after they complete their production process and earn revenues. The profit for the cash unconstrained firms is given below:

⁵For an extension with different productivity across firms see Section 7.

$$\Pi_{u,t} = p_{u,t}\theta_t H_{u,t} - w_t H_{u,t}. \quad (5)$$

The cash constrained firms, after realizing the productivity and monetary shocks, decide to borrow $b_{d,t}$ amount of cash from the loanable funds market in order to hire H_t^c amount of labor. These firms need to acquire cash in advance in order to start their production process; they pay the competitive wage w_t upfront for hiring workers. After they produce and sell their goods, the cash constrained firms repay their loans with net nominal interest R_t . The profit for the cash constrained firms is as follows:

$$\Pi_{c,t} = p_{c,t}\theta_t H_{c,t} - w_t H_{c,t} - R_t b_{d,t}. \quad (6)$$

The cash constrained firms' profit maximization problem is subject to a cash-in-advance constraint, $w_t H_{c,t} \leq b_{d,t}$, so that the amount of loanable funds demanded needs to cover at least the labor cost of the cash constrained firms. The first order conditions imply that $R_t = \zeta_t$ and $p_{c,t}\theta_t = w_t(1 + \zeta_t)$, where ζ_t is the multiplier of the constrained firms' cash-in-advance constraint.

After the production process takes place and profit is determined, both types of firms pay profits to the household. The firms' profit maximization conditions are as follows:

$$p_{c,t}\theta_t = w_t(1 + R_t), \quad (7)$$

$$p_{u,t}\theta_t = w_t, \quad (8)$$

which imply that:

$$p_{c,t} = (1 + R_t)p_{u,t}. \quad (9)$$

Condition (7) reveals that, for the cash constrained firms, the nominal interest rate drives a wedge between the marginal product of labor and the competitive real wage. Alternatively, the cost per unit of labor is higher than the real wage, and it crucially depends on the interest rate at which constrained firms borrow from the loanable funds market. As condition (7) clearly shows, this effect does not exist for the unconstrained firms. Furthermore, condition (9) suggests that the nominal interest rate creates a wedge between the price of the two goods.

The combination of conditions (4) and (9) yields:

$$\frac{\mathcal{U}_{c,t}}{\mathcal{U}_{u,t}} = 1 + R_t, \quad (10)$$

which shows that the MRS is a function of the nominal interest rate. A decline in R_t reduces $\frac{\mathcal{U}_{c,t}}{\mathcal{U}_{u,t}}$, suggesting that the consumption of the product of constrained firms rises relative to the consumption of the product of unconstrained firms. This is the first result that we establish about the role of monetary policy in tilting consumption across sectors. In what follows, we show how monetary policy leads to changes in the allocation of labor across sectors.

Since the production functions are given by $c_{c,t} = \theta_t H_{c,t}$ and $c_{u,t} = \theta_t H_{u,t}$, the ratio of labor of the constrained firms to labor of the unconstrained firms equals the ratio of the corresponding consumption levels:

$$\frac{c_{c,t}}{c_{u,t}} = \frac{H_{c,t}}{H_{u,t}}, \quad (11)$$

Then, using condition (10), we may write the relative labor as:

$$\frac{H_{c,t}}{H_{u,t}} = f'(R_t), \quad (12)$$

with $f' < 0$. Therefore, a decline in the nominal interest rate reduces not only the relative consumption of the constrained firms but also their relative labor. This is the key theoretical result that we obtain from the analysis of this section; it illustrates the role of monetary policy in distributing resources across sectors.

2.3 Market Clearing and Monetary Policy

We will look at an equilibrium where prices w_t and $p_{c,t}$, $p_{u,t}$ are strictly positive, and n_t , $H_{c,t}$, $H_{u,t}$ are interior and bounded between zero and one. Also, the multipliers are strictly positive. Binding cash-in-advance constraint implies:

$$\frac{m_{t-1}}{1 + \pi_t} - n_t + w_t H_t = p_{c,t} c_{c,t} + p_{u,t} c_{u,t}. \quad (13)$$

There are four markets in the model. The market clearing conditions are as follows:
The goods markets clear:

$$c_{c,t} = \theta_t H_{c,t}, \quad (14)$$

and

$$c_{u,t} = \theta_t H_{u,t}. \quad (15)$$

The labor market clears:

$$H_{c,t} + H_{u,t} = H_t. \quad (16)$$

The loanable funds market clears:

$$w_t H_{c,t} = n_t + \tau_t. \quad (17)$$

The monetary authority affects the nominal interest rate through a Taylor-type rule. Using the money market, the cash that the monetary authority injects into the economy, τ_t is given by: $\bar{m}_t = \frac{\bar{m}_{t-1}}{1+\pi_t} + \tau_t$. The money market clears:

$$m_t = \bar{m}_t. \quad (18)$$

This concludes the description of the model.

2.4 Functional Forms

We assume Constant Elasticity of Substitution (CES) utility function for consuming the two types of goods and convex disutility function of labor. The period utility function of the family is given by:

$$\mathcal{U}(c_{c,t}, c_{u,t}) - D(H_t) = \left[\kappa c_{c,t}^{\frac{\xi-1}{\xi}} + (1-\kappa) c_{u,t}^{\frac{\xi-1}{\xi}} \right]^{\frac{\xi}{\xi-1}} - \alpha \frac{H^{1+\eta}}{1+\eta}, \quad (19)$$

with ξ being the elasticity of substitution between the good produced by constrained firms and the good produced by unconstrained firms, κ is the share of consumption of constrained firms out of total consumption, η is the intertemporal elasticity of substitution of labor supply and α is a scaling parameter that measures the disutility from supplying labor.

Given the CES utility function the corresponding aggregate price level index reads:

$$P_t = \left[\kappa P_{c,t}^{1-\xi} + (1-\kappa) P_{u,t}^{1-\xi} \right]^{\frac{1}{1-\xi}}. \quad (20)$$

Also, using the specified functional forms, conditions (10) and (14)-(15) yield:

$$\frac{H_{c,t}}{H_{u,t}} = \left(\frac{\kappa}{1-\kappa} \right)^{\xi} \left(\frac{1}{1+R_t} \right)^{\xi}. \quad (21)$$

This condition well captures the effects of monetary policy on the allocation of labor across sectors. Other things equal, a rise in the interest rate leads to a decline in the relative amount of labor employed by the constrained firms. This effect depends on the size of the elasticity of substitution between the products of the two types of firms (ξ): the larger ξ , the stronger this channel is, and the more responsive is relative employment to changes on

the interest rate.⁶

3 Parameterization

We assume a time unit of one quarter. We set $\beta = 0.99$, implying an annual real interest rate of roughly 4%, as is conventional in the literature. We set $\eta = 0.5$ so that the labor supply elasticity is within the traditional levels in macroeconomic models and to help capture the volatility of total hours in this model that does not account for labor market frictions. In each experiment we run, α will be set so that the steady state value of hours worked relative to total hours (H) is 0.21, capturing an average workweek of 35 hours (out of 168), which is consistent with its historical average in the United States. Produced through the model-based empirical evidence that we provide in Section 4, we set $\kappa = 0.353$ and $\xi = 1.4$. We then provide results using alternative values of these two parameters in the robustness analysis, Section 6.

There are two shocks in the model economy: total factor productivity (TFP) shock and monetary policy shock. The exogenous process of TFP is given by:

$$\ln \theta_t = (1 - \rho_\theta) \ln \theta + \rho_\theta \ln \theta_{t-1} + \epsilon_{\theta,t}, \quad (22)$$

with $\theta = 1$, autocorrelation $\rho_\theta = 0.90$ and a standard error $\sigma_\theta = 0.02$.⁷

Monetary policy is governed by the following Taylor-type rule:

$$\ln \left(\frac{1 + R_t}{1 + R} \right) = \phi_R \ln \left(\frac{1 + R_{t-1}}{1 + R} \right) + (1 - \phi_R) \left[\phi_\pi \ln \left(\frac{1 + \pi_t}{1 + \pi} \right) + \phi_y \ln \left(\frac{y_t}{y} \right) \right] + \varepsilon_{r,t}, \quad (23)$$

with y_t being the total production (output) in the economy and the undated variables denoting the steady state values of their respective variables. We set the standard values $\phi_\pi = 1.5$ and $\phi_y = 0.5/4$ for the coefficients of inflation and output respectively, in the monetary policy rule. Also, $\varepsilon_{r,t}$ is the monetary policy shock and $\phi_R = 0.9$ is the interest-rate smoothing parameter that is set to match the persistence of the nominal interest rate. In what follows, we will also consider the case of $\phi_R = 0$ (see Section 6). In each exercise, the standard deviation of the shock is set to match the standard deviation of the U.S. gross domestic product. Here, the standard deviation of the shock is $\sigma_r = 0.06$.

⁶In the extreme case of perfect substitutes ($\xi \rightarrow \infty$) and given that the interest rate is positive (or non negative), the relative amount of labor will approach zero if $\kappa < 0.5$. In what follows we show that indeed this is the case. Intuitively, when the goods are perfect substitutes in households' preferences, only one of these goods will ultimately be produced; in this case, it is the product of the unconstrained firms.

⁷The value of σ_θ is chosen so to match the average volatility of US gross domestic product, i.e., we let standard deviation of US output be 0.015.

4 Empirical Analysis

4.1 Overview

In this section, we provide empirical support regarding the effects of interest rate changes on the employment shares of constrained and unconstrained firms. From our estimation we will get information about the values of the two key parameters of the model, κ and ξ . As explained earlier, we follow previous literature and consider small firms as constrained, and large firms as unconstrained.

The classification of firms as “small” in the literature varies, but it mostly corresponds to firms with up to 19 employees (as, for example, in [Fort et al., 2013](#)) or firms with up to 49 employees (e.g. [Moscarini and Postel-Vinay, 2012](#)). For this reason we base our analysis on both definitions. Given that our model accounts for two sectors only, the mapping between the relative employment of the constrained firms in the model ($H_{c,t}/H_{u,t}$) and the data, requires us to use all other firms (medium and large) as the alternative. A caveat here is that firms with 20-499 employees or 50-499 employees, which are thought as medium sized, are categorized as large firms in our analysis, i.e., they are the unconstrained firms. In the robustness analysis (Section 6) we discuss quantitative results that are produced using the classification of [Carlstrom et al. \(2010\)](#) who considered “small firms” as employing up to 499 workers and large firms otherwise.

Our sample covers the period 1993:Q1-2014:Q2 due to the availability of quarterly data on the distribution of employment by firm size, given by the United States Census Bureau. To account for the cost of borrowing by small firms, we use the interest rate on borrowing that is provided by the National Federation of Independent Businesses (NFIB). The typical NFIB member employs 10 people. The data sample starts in 1986. This interest rate reflects the average cost of borrowing of small businesses as have been reported by these businesses to the NFIB. We use this series for two reasons. First, it better captures the cost of borrowing of small businesses than the policy interest rate or the interest rates on short-term treasury bonds. Second, the federal funds rate and the interest rates on short-term treasury securities have been at the zero-lower bound between late 2008 and late 2015. However, the actual cost of borrowing of firms has not been zero. To overcome this shortcoming and to avoid ending the sample in 2008:Q3, we use the NFIB’s series.⁸

⁸We have also performed the same exercise with a different interest rate, the Moody’s Seasoned Corporate Bond Yield (BAA), and the main conclusions of the paper do not change. Namely, the rise in the interest rate leads to a decline in the relative employment of small firms.

Figure 1: The cyclical components of relative employment of small firms (over total employment) and the cost of borrowing of small firms. Data sources: NFIB (interest rate) and United States Census Bureau (Employment). Note: annual averages. The data are HP-Filtered with a smoothing parameter of 1600.

Figure 1 shows the co-movement between the relative employment of small firms and the above mentioned interest rate. For most of the sample period, the two series exhibit a clear negative correlation: a correlation coefficient of -0.33 when firms with up to 19 employees are considered small, and a correlation coefficient of -0.23 when small firms are defined as having up to 49 employees. Therefore, from Figure 1 we conclude that a rise in the cost of borrowing is associated with a decline in the employment of small firms relative to employment of large firms.

4.2 Econometric Analysis

We turn to formally test whether an increase in the cost of borrowing of small firms reduces their relative employment. In addition, we can estimate the parameters ξ and κ by making use of condition (21). Taking natural logarithms on both sides of this condition gives:

$$\ln \left(\frac{H_{c,t}}{H_{u,t}} \right) = \xi \ln \left(\frac{\kappa}{1 - \kappa} \right) - \xi \ln (1 + R_t) . \quad (24)$$

To estimate this equation, we use the above mentioned data on employment and interest rate. We conduct our analysis using both Ordinary Least Squares (OLS) and Generalized Methods of Moments (GMM) with instrumental variables (IVs). A clear advantage of this analysis is that the slope of the gross interest rate is equal to ξ , for which we have no prior information. In this respect, ξ shows up as the elasticity of the relative employment with respect to changes in the cost of borrowing.

In line with existing studies in macroeconomics (e.g. [Gali and Gertler, 1999](#), [Murray,](#)

2006, Ravenna and Walsh, 2006, and Gali et al., 2007) we use the lags of variables as instruments. As West et al. (2009) point out, the basis for using the lags of variables as instruments is that, usually, if a variable is a legitimate instrument, then so are lags of that variable. Using the lags of the variables helps with overcoming the possibility of endogeneity and bi-directional causality: the current value of the dependent variable (relative employment of small firms) is unlikely to affect the past values of the independent variable (the cost of borrowing of firms). As instruments, we use four lags of the interest rate, output gap and the unit labor cost (which measures the average cost of labor per unit of output). The output gap is added to account for the possibility of co-movement of both the relative employment and interest rate over the business cycle, and the unit labor cost is added to control for the overall costs of firms.

	OLS		GMM	
Firm Size	0-19	0-49	0-19	0-49
<i>Const.</i>	-1.118*** (0.045)	-0.571*** (0.043)	-1.024*** (0.058)	-0.488*** (0.060)
$\ln(1 + R_t)$	-1.101** (0.442)	-0.923** (0.431)	-1.936*** (0.602)	-1.653*** (0.628)
κ	0.266	0.350	0.371	0.427
<i>Adj. R²</i>	0.716	0.776		
<i>J-Stat</i>			10.419	10.490
<i>P-Value</i>			(0.405)	(0.3986)
<i>N</i>	86	86	86	86

Table 1: Dependent variable: natural logarithm of the relative employment of small firms (H_t^c/H_t^u). Results of estimating condition (24). Standard errors in parentheses. * significant at the 10% level, ** significant at the 5% level, *** significant at the 1% level. *J-Stat* is the statistic for the hypothesis that the over-identifying restrictions are satisfied. IVs for GMM estimation: four lags of the interest rate, output gap and unit labor cost. Time trend included. Sample period: 1993:1-2014:2.

The estimation results for firms up to 19 employees and up to 49 employees are summarized in Table 1. In all cases, the coefficient of $1 + R_t$ is negative and significant, so that an increase in the interest rate reduces the relative employment of small firms. Also, in most cases, the coefficient is above 1 in absolute value, suggesting that there exists more than one-for-one relationship between changes in the interest rate and the relative employment. The analysis also reveals that for firms with 0-49 employees the response is weaker than it is for firms with 0-19 employees. This finding suggests that the smallest firms are the ones affected the most by interest rate changes.

Given the lack of conjecture on what constitutes a small firm and the exact number of workers that a small firm employs, we use the average of the coefficients that the two

definitions produce. As shown in Table 1, the average coefficient of $1 + R_t$ with OLS is -1.012 . With GMM, the average coefficient, in absolute value, is higher, -1.79 .⁹ For our baseline exercise we use the average across the results of OLS and GMM and both definitions, and therefore we set $\xi = 1.4$.

The bottom row of the table presents the implied value of κ . Similar to the choice of ξ , we produce our baseline parameter for $\kappa = 0.353$ using the average of the four columns of Table 1. In the robustness analysis, we use an alternative value of this parameter as well as an alternative value of the parameter ξ . We also report the results of an innovation to κ , which can be seen as a change in consumer preferences.

Figure 2: The steady-state relative labor of the constrained firms as a function of the elasticity of substitution parameter ξ .

Given the benchmark value of κ , we use condition (21) to obtain the steady-state relative level of labor for different levels of substitutability, i.e., for different values of the parameter ξ . We consider the range 1.4-9.6, which is between the benchmark value of this parameter and 7 times the benchmark value. The results are shown in Figure 2. The share of labor for the constrained firms declines rapidly when ξ rises and it approaches zero for sufficiently high values of ξ . Therefore, using a very large value of ξ will render the differential response of firms as almost irrelevant because the constrained-firm sector becomes negligible. In addition, the case of perfect substitutes can be safely approximated by a value of ξ within the range that we consider here, and thus extreme values (or infinity)

⁹For all GMM specifications, the J Test indicates that the over-identification restrictions are satisfied.

are not necessary to capture that case. Figure 2 implies that for our analysis to hold, we need elasticity of substitution between 1 and 6. Our estimates of 1.4 is well within this range.

5 Quantitative Results

We perform a quantitative exercise in order to assess the importance of firms heterogeneity in their response to productivity and monetary shocks. The results are presented in Figures 3 and 4. Figure 3 shows the effects of a decrease in the nominal interest rate through the Taylor-type rule. All variables, with the exception of the nominal interest rate, are in natural logarithms. Expansionary monetary policy induces a rise in the production and employment of both types of firms and, therefore, implies higher total output and employment. Consistent with condition (21), due to the decrease in the interest rate, the cost of external financing of the constrained firms declines, which leads to a rise in their relative employment. The response of constrained firms to changes in the nominal interest is clearly larger than the response of unconstrained firms, suggesting that the constrained firms are more cyclical after monetary policy shocks.¹⁰

Figure 4 presents the response of the economy to a positive TFP shock. The improvement in technology leads to a rise in the production and labor of both sectors. As implied by the Taylor rule, the rise in output induces a rise in the nominal interest rate. In turn, the rise in the nominal interest rate lowers the labor of the constrained firms relative to the labor of the unconstrained firms. Therefore, while the rise in productivity is favorable for both types of firms, unconstrained firms benefit from the improved technology more: since unconstrained firms do not rely on external funds, they can hire labor easier than constrained firms, inducing a decrease in the relative employment of constrained firms. Put differently, the advancement in technology disproportionately favors the firms that do not need to pay the external finance premium, namely the unconstrained firms.

A comment on the response of relative employment to a productivity shock is in order: since all firms have the same productivity (θ_t), the decline in $\frac{H_{c,t}}{H_t}$ results only from the rise in the interest rate. If the interest was fixed (as may occur at the zero lower bound), $\frac{H_{c,t}}{H_t}$ would not have changed (see condition (21)). For this reason, Section 7 discusses the case where the productivity levels of constrained and unconstrained firms are allowed to differ.

¹⁰The behavior of $\frac{H_{u,t}}{H_t}$ is not a mirror image of the behavior of $\frac{H_{c,t}}{H_t}$ because both variables are presented in natural logarithms. This can be seen by noting that: $\ln(\frac{H_{u,t}}{H_t}) = \ln(1 - \frac{H_{c,t}}{H_t})$. This presentation will be maintained throughout the entire paper.

Figure 3: Percentage deviations from steady state after a one standard-deviation positive monetary policy shock for the constrained firms production, unconstrained firms production, total employment, employment of cash constrained firms, employment of unconstrained firms, relative employment of cash constrained firms, relative employment of unconstrained firms, cost of external financing and nominal interest rate. Note: $\phi_R = 0.9$.

Table 2 shows the volatility of key variables relative to the volatility of aggregate output. A shock to the nominal interest rate generates considerably more volatility in the production of the constrained firms than a shock to TFP does (2.642 vs. 0.985). And, an interest rate shock generates four times more volatility in the labor of the constrained firms compared to a TFP shock (2.642 vs. 0.649). On the other hand, unconstrained firms are considerably more reactive to TFP shocks than to monetary policy shocks: a TFP shock generates three times more output volatility than what a monetary policy shock does (1.007 vs. 0.305) and twice more employment volatility (0.670 vs. 0.305). These findings largely align with the empirical evidence that small firms react sharply to monetary shocks (Gertler and Gilchrist, 1994; Chari et al., 2013; Kudlyak and Sanchez, 2016) but are less cyclical than unconstrained firms during a typical business cycle (e.g., Chari et al., 2013; Moscarini and Postel-Vinay, 2012).¹¹ Our results indicate a very important and noticeable difference

¹¹Specifically, Moscarini and Postel-Vinay (2012) look at employment growth and find that the differential growth of employment between large and small firms covaries positively with GDP growth. Our findings hold for growth employment differentials too. That is, the model implies that the differential growth of

Figure 4: Percentage deviations from steady state after a one standard-deviation positive productivity shock for the constrained firms production, unconstrained firms production, total employment, employment of cash constrained firms, employment of unconstrained firms, relative employment of cash constrained firms, relative employment of unconstrained firms, cost of external financing and nominal interest rate. Note: $\phi_R = 0.9$.

between monetary policy shocks and TFP shocks: while monetary policy shocks induce a strong response in the employment and production of the constrained firms, productivity shocks have considerably smaller effects on the redistribution of labor across firms. This occurs because the cost of external financing is strongly affected by monetary policy and only mildly affected by productivity shocks.¹² For this reason, productivity shocks are less redistributational than monetary policy shocks.

The difference between monetary policy shocks and TFP shocks in affecting the distribution of labor across firms is well illustrated by Table 2. From the last column we see that the volatility produced in the relative employment of cash constrained firms is much higher after a monetary shock than after a productivity shock (1.642 vs. 0.015). These results align with the implications of empirical findings suggesting that monetary policy shocks

employment between cash unconstrained and constrained firms increases after a positive productivity shock.

¹²Empirically, most interest rates are strongly affected by monetary policy (see for example Kuttner, 2001). The effects of productivity shocks are more controversial as they depend on the circumstances of the specific recession, the response of monetary policy to the surge in economic activity, etc.

create large differences across firms' responses, with small firms being more responsive, although looking at NBER business cycle dates the differences are smaller, with large firms being more responsive (see [Chari et al., 2013](#)).

	y_c	y_u	H_c	H_u	H	H_c/H	H_u/H
Monetary Policy Shock	2.642	0.305	2.642	0.305	1.000	1.642	0.695
Productivity Shock	0.985	1.007	0.649	0.670	0.650	0.034	0.015

Table 2: Model-based relative standard deviations. Note: all variables are in natural logarithms.

Our focus is on the differential effects of the shocks on the relative employment of cash constrained and unconstrained firms. However, our model produces responses of aggregate variables as well, which we can compare to earlier work and quantitative models. [Christiano et al. \(1996\)](#) find that a roughly 75 basis points decrease in the federal funds rate produced by a VAR monetary policy shock implies a roughly 65 basis points increase in real GDP, 45 basis points increase in employment and 45 basis points increase in prices (commodity prices).¹³ In our exercise, 75 basis points decrease in the nominal interest rate is associated with 44.5 basis points increase in total output and employment, and a 73.57 basis points increase in inflation.

For the productivity shock we compare our findings to those of the business cycle model of [King and Rebelo \(1999\)](#).¹⁴ In [King and Rebelo \(1999\)](#), a productivity shock that increases output by about 1.5% increases labor input by 0.7% and the real interest rate by 22 basis points. In our model, an increase in productivity that increases output by 1.5%, increases total employment by almost 1%, and the real interest rate by 7.3 basis points. Overall, our results match the findings of earlier work qualitatively. Furthermore, although we compare results from very different models, most of our results are also close to previous work quantitatively.

6 Robustness Analysis

In this section we conduct the following robustness analysis: i) we consider alternative values for the parameters ϕ_R , κ and ξ , and ii) we evaluate the effects of a shock to the

¹³[Christiano et al. \(1996\)](#) use various methods but produce similar results. We use the first row of Figures 3 and 4 of their study, from which we roughly infer the maximum effects of the policy shock. Note that they show the effects of an increase in the federal funds rate while we report the effects of a decrease in the federal funds rate.

¹⁴We use [King and Rebelo \(1999\)](#)' Figure 11 from where we roughly infer the maximum effects of the productivity shock.

parameter that governs the disutility of labor (α) and the weight of the constrained firms' output in the utility function (κ). The results are presented in Appendix B.

6.1 Alternative Parameter Values

We first present the results of our quantitative exercise without interest-rate smoothing in the Taylor-type of rule ($\phi_R = 0$). Figure B.1 shows that an expansionary monetary policy shock leads to a rise in the production and labor of both types of firms. Key to our analysis is that labor of the constrained firms rises relatively to the labor of unconstrained firms. However, these effects are short-lived and the system reverts back to the initial steady state quickly. A shock to TFP has similar implications to the shock to TFP with interest rate smoothing, but with some quantitative differences (Figure B.2). In either case, this analysis supports our findings regarding the implications of monetary policy and TFP shocks for the distribution of labor across sectors that differ in their financing schemes.

We next consider a higher value of κ ; in particular, we set $\kappa = 0.53$ to match the average share of firms of up to 499 employees (implying that we consider larger firms than before for the small firms category, in line with Carlstrom et al., 2010). All other parameters remain as in the benchmark analysis. Figures B.3 and B.4 present the impulse response functions for a monetary policy shock and a TFP shock, respectively. Overall, the response of the economy to both shocks is similar to the benchmark analysis. In particular, expansionary monetary policy stimulates economic activity and leads to a rise in the relative employment of the constrained firms. A rise in TFP induces a rise in output and labor of both types of firms, but the labor of the constrained firm falls relative to the labor of the unconstrained firms. In addition, as in the benchmark analysis, the effects of TFP on the redistribution of labor is muted compared to the effects of a monetary policy shock that directly affects the relative employment of the constrained firms.

We then consider a higher value of $\xi = 5.6$ (four times the benchmark value) which, through condition (21) implies a stronger response of the relative employment of constrained firms to changes in the cost of borrowing. The results for a monetary policy shock and a TFP shock are shown in Figure B.5 and Figure B.6, respectively (all other parameters, including κ , are as in the benchmark case). Our analysis confirms that the production of constrained firms as well as the relative amount of labor of these firms respond stronger than in the benchmark case, particularly following a monetary policy shock (notice the very strong response of relative employment to a monetary policy shock). In this respect, the substitutability between the two types of products is an important factor in determining the response of each sector to exogenous shocks. Furthermore, the results of this subsection

indicate that our main findings are robust to changing this key parameter of the model.¹⁵

6.2 Labor Disutility and Preference Shocks

In this subsection we show the results of two experiments. First, we compute the response of constrained and unconstrained firms to changes in the disutility of work (α), which we think of as a shift in the supply of work. Second, we show the response of firms to changes in the preferences of households over output of the constrained firms vs. output of the unconstrained firms (namely, a change in κ). We label a shock to κ as a preference shock.

As we see from Figure B.7 in Appendix B, a decline in α induces a rise in labor supply and total labor (H_t) in equilibrium. The increase in the total amount of labor is split between both types of firms, leading to a rise in labor and production of both types of firms. The rise in economic activity renders the interest rate higher, increases the external financing cost and, thus, reduces the relative employment of the constrained firms. In this respect, the shift in labor supply is met with a higher increase in labor demanded by the unconstrained firms (compared to the constrained ones), which induces a rise in their relative employment.

On the other hand, Figure B.8 shows that if consumers' preferences suddenly shift towards the output of constrained firms, then labor and production of these firms will rise while labor and production of the unconstrained firms will decline. Consequently, the relative amount of labor of constrained firm rises dramatically (because we have a simultaneous rise in $H_{c,t}$ and a decline in $H_{u,t}$). The increase in labor demand from the constrained firms increases the demand for loanable funds and in turns increases the nominal interest rate. With the exception of this scenario (a shift towards the good of the constrained firms) that clearly affects the demand for loanable funds, a rise in the nominal interest rate is associated with unfavorable effects on the share of labor of the constrained firms, as we show in all previous experiments.

7 Different Productivity Levels

Our analysis so far assumed that constrained and unconstrained firms have the same access to technology and, as a result, the same level of productivity. We now modify this

¹⁵When we increase this parameter to $\xi = 7$, which is five times the benchmark, we get an extremely large response to a monetary policy shock. Also, with $\xi = 7$ or higher, actual (and not only relative) labor of the constrained firms declines following a positive productivity shock, which in turn reduces the output of these firms. This occurs because, with sufficiently high values of ξ , the negative interest rate effect on the labor of the constrained firms outweighs the positive effect of higher productivity.

assumption by letting the two types of firms have different levels of productivity. The key motivation behind this exercise is that small firms are likely to have worse access to newer technologies. Letting $\theta_{c,t}$ and $\theta_{u,t}$ be the productivity of the constrained and unconstrained firms respectively, the solutions to the profit-maximization problems of the two types of firms yield:

$$p_{c,t}\theta_{c,t} = w_t(1 + R_t), \quad (25)$$

$$p_{u,t}\theta_{u,t} = w_t, \quad (26)$$

which imply the following:

$$p_{c,t} = (1 + R_t)p_{u,t} \frac{\theta_{u,t}}{\theta_{c,t}}. \quad (27)$$

Since we allow for movements of labor between sectors, we are assuming that both types of firms are paying the same wage w_t . Then, using the specified functional forms we have:

$$\frac{H_{c,t}}{H_{u,t}} = \left(\frac{\kappa}{1 - \kappa} \right)^\xi \left(\frac{\theta_{c,t}}{\theta_{u,t}} \right)^{\xi-1} \left(\frac{1}{1 + R_t} \right)^\xi. \quad (28)$$

This modification does not alter the observation that the cost of borrowing negatively affects the relative employment of constrained firms. Yet, it introduces the ratio of productivity levels as a factor affecting the distribution of labor across firms. In particular, the relative amount of labor employed by the constrained firms is positively related to their relative productivity. In addition, since $\xi > 1$, this condition shows that the constrained firms' relative employment is more sensitive to interest rate changes than what it is to changes in the the relative productivity. We explore the effects of productivity and monetary policy shocks allowing for productivity differences.

7.1 Effect of Productivity Shocks

We consider two cases. First, the productivity of the unconstrained firms improves while that of the constrained firms remains unchanged (and both types of firms have the same steady-state productivity levels). Second, the steady-state productivity of the constrained firms is lower than the steady-state productivity of the unconstrained firms and it changes slower than the productivity of the unconstrained firms (i.e. the rise in the productivity of constrained firms following an improvement in technology is smaller).

For the second experiment, we calculate the difference between the average productivity of small and large firms using U.S. data, with average productivity being measured as the ratio of receipts (revenues) to employment. This definition is consistent with our production function, namely $\theta_{c,t} = y_{c,t}/H_{c,t}$ and $\theta_{u,t} = y_{u,t}/H_{u,t}$. The data are available for 1997, 2002

and 2007, and they yield an average difference of 39% over this period. Therefore, for the second case we have: $\theta_{c,t} = 0.61\theta_{u,t}$.

Figure 5: Percentage deviations from steady state after a one standard-deviation positive productivity shock for the constrained firms production, unconstrained firms production, total employment, employment of cash constrained firms, employment of unconstrained firms, relative employment of cash constrained firms, relative employment of unconstrained firms, cost of external financing and nominal interest rate.

Figure 7 shows the results of these two exercises and the benchmark case. The response of the production and employment of both types of firms is milder compared to the benchmark case. The difference is sharper for the constrained firms. The increase in constrained firms' output is reduced by nearly a third when $\theta_{c,t}$ is constant and by two thirds when $\theta_{c,t}$ rises by less than $\theta_{u,t}$. Note that the relative employment of the constrained firms declines more under the two new cases compared to the benchmark, despite smaller difference in the reaction of the interest rate. This occurs because under the two experiments that we consider here, changes in productivity have direct effects on the distribution of labor, regardless of the nominal interest rate, as equation (28) makes clear.

Therefore, if the two types of firms differ in their access to technology, a positive productivity shock will benefit the unconstrained firms more than the constrained ones, and the difference between them is amplified compared to the benchmark case. Now, there is a clear difference in the response of the two types of firms, making the unconstrained firms considerably more cyclical than the constrained ones when it comes to aggregate (productivity) shocks. However, despite this increase in the response of relative employment to productivity shocks, the response remains smaller than the corresponding response to an interest rate shock (as documented in Figure 3), which supports our earlier finding that monetary policy has a bigger distributional role than changes in aggregate productivity.

Figure 6: Percentage deviations from steady state after a one standard-deviation positive productivity shock for the constrained firms production, unconstrained firms production, total employment, employment of cash constrained firms, employment of unconstrained firms, relative employment of cash constrained firms, relative employment of unconstrained firms, productivity of constrained firms and productivity of unconstrained firms. Note: constant nominal interest rate and different productivity levels.

The model with different productivity levels allow us to consider an interesting case regarding monetary policy: the behavior of the economy with a constant nominal interest rate (as might occur at the zero lower bound). When the nominal interest rate is constant, condition (28) implies that relative employment will change only if the relative productivity of the constrained firms adjusts. We now present the findings when the nominal interest

rate is set at its steady state value ($R_t = (1/\beta) - 1 \forall t$) and let the productivity of the unconstrained firms react sluggishly to improvements in technology.

Our findings are summarized in Figure 6. As before, an improvement in technology induces higher production and labor for both types of firms. However, the bigger rise in the productivity of the unconstrained firms renders their relative labor higher. In this case, the behavior of the interest rate is not the driving force behind the result (as the interest rate is irresponsive to the shock). Rather, on impact, the muted reaction of the constrained-firms' productivity is disadvantageous for these firms. We thus conclude that the adoption of a Taylor-type rule in the benchmark analysis is not the only reason for our findings regarding the reaction of relative labor to changes in productivity.

7.2 Effect of Monetary Policy Shocks

We examine the effect of monetary policy shocks when the steady-state productivity of the constrained firms is lower than the steady-state productivity of the unconstrained firms. We compare this case with the benchmark case (where both types of firms have the same steady state productivity).¹⁶

As Figure 7 shows, monetary policy has almost identical effects on the labor and production of the constrained firms, no matter how productive they are. That is, expansionary monetary policy increases the labor and production of these firms in roughly the same extent regardless of their productivity levels. However, the lower productivity of the constrained firms reflects on aggregate variables, as total labor and production increase by less than in the benchmark case. Then, the less productive the constrained firms are, the less will a monetary policy expansion benefit the unconstrained high-productivity firms; in other words, monetary policy is shifting resources from more productive firms to less productive firms. Therefore, while aggregate output rises following a monetary expansion, the rise in aggregate output is smaller if the constrained firms are less productive than the unconstrained firms; monetary policy is less effective in stimulating economic activity than in the benchmark analysis.¹⁷

¹⁶We do not examine the case of a constant productivity of the constrained firms because it will essentially be similar to the benchmark case, as the shock is to monetary policy and not productivity.

¹⁷Note that, given the positive external finance premium, the constrained firms stop producing at marginal product of labor higher than the unconstrained firms; namely, with lower than intended labor. A decrease in the interest rate in that sense is always a good idea in the way the term is used in the misallocation literature (see for example in [Banerjee and Moll, 2010](#)). This is because, although the constrained firms now have lower productivity, they still have higher marginal product of labor compared to the unconstrained ones.

Figure 7: Percentage deviations from steady state after a one standard-deviation monetary policy shock for the constrained firms production, unconstrained firms production, total employment, employment of cash constrained firms, employment of unconstrained firms, relative employment of cash constrained firms, relative employment of unconstrained firms, cost of external financing and nominal interest rate.

8 Conclusions

We use a model with heterogenous firms that differ in their financing schemes and, therefore, in their labor hiring decisions. The cash constrained firms rely on external funding and are subject to external finance premium while the cash unconstrained ones finance activities internally. Because of the dissimilarity in the way the two types of firms finance operations, productivity and monetary policy shocks affect them differently.

Our analysis indicates that cash constrained firms are more cyclical compared to unconstrained firms following monetary policy shocks. In this respect, monetary policy is playing a crucial role in the distribution of labor, and by extension in the distribution of economic activity, across sectors that differ in their financing schemes. On the other hand, the cash unconstrained firms are more cyclical than the constrained ones after productivity

shocks. One implication of this finding is that the rise in the relative employment of the constrained firms following recessionary productivity shock can be seen as consistent with the implications of empirical findings (e.g. [Moscarini and Postel-Vinay, 2012](#) and [Haltiwanger et al., 2013](#)) and the popular view that small/young firms are generally the engine of job creation during recessions.

Comparing the distributional effects of the two shocks, our analysis indicates that the effects of a change in productivity on the distribution of labor are considerably smaller than the effects of monetary policy. The latter remains the main force in creating disparities between firms, emphasizing an important role of monetary policy in redistributing resources. The role of monetary policy in allocating resources across sectors, however, may lead to undesirable consequences: when the model allows for differentiated levels of productivities, with the constrained firms being less productive, a monetary policy expansion tilts resources towards the less productive firms and away from the more productive firms. In this case, the rise in aggregate output is smaller than the rise in aggregate output when both types of firms are equally productive, implying that monetary policy is less effective in stimulating the economy.

We have argued that our results are consistent with the implications of empirical literature analyzing the effects of an average US recession or tight money episode. There are conflicting views, however, regarding the Great Recession. [Fort et al. \(2013\)](#), taking into account firm size and age find that small/young firms were particularly affected by the Great Recession. [Giroud and Mueller \(2015\)](#) show that establishments of high-leverage firms, who act like financially constrained firms, experience a significantly larger decline in employment between 2007 and 2009 compared to establishments of low-leverage firms. However, [Kudlyak and Sanchez \(2016\)](#) find that it was large firms that responded more to the last recession in terms of sales and short term debt. One difference is the definition of small firms, which is in terms of employment in the former cited literature and in terms of assets in the latter. Another issue is that the firms' behavior during the Great Recession could be coming as a response to a tight money shock, other adverse shocks that lead to the recession, or a combination of the two, as the latter authors have argued. In this case, our model would not have clear prediction regarding the Great Recession, and results would depend on the relative strength of the two shocks.

Our work identifies a new channel through which monetary policy affects the economy, namely through the reallocation of labor across firms. This result motivates future empirical research in order to identify differential effects of shocks on heterogeneous firms' production and employment, and the reallocation of resources across firms following these shocks.

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A Household's Problem

The family solves:

$$v(m_{t-1}) = \max_{c_{c,t}, c_{u,t}, H_t, n_t, m_t} [\mathcal{U}(c_{c,t}, c_{u,t}) - D(H_t) + \beta E_{\{\theta_{t+1}, \varepsilon_{t+1}\}} v(m_t)], \quad (\text{A.1})$$

under the constraints 2 and 3 :

$$\frac{m_{t-1}}{1 + \pi_t} - n_t + w_t H_t \geq p_{c,t} c_{c,t} + p_{u,t} c_{u,t},$$

and

$$\frac{m_{t-1}}{1 + \pi_t} + w_t H_t + R_t n_t + (1 + R_t) \tau_t + \widetilde{\Pi}_t \geq m_t + p_{c,t} c_{c,t} + p_{u,t} c_{u,t}.$$

Let $\gamma_t, \delta_t \geq 0$ be the multipliers for equations (2) and (3) respectively. The first order conditions for the household's maximization problem are given bellow:

With respect to n_t :

$$- \gamma_t + \delta_t R_t = 0, \quad (\text{A.2})$$

with respect to $c_{1,t}$:

$$\mathcal{U}_{c,t} - p_{c,t} \gamma_t - p_{c,t} \delta_t = 0, \quad (\text{A.3})$$

with respect to $c_{2,t}$:

$$\mathcal{U}_{u,t} - p_{u,t} \gamma_t - p_{u,t} \delta_t = 0, \quad (\text{A.4})$$

with respect to H_t :

$$- D'(H_t) + w_t \delta_t + w_t \gamma_t = 0, \quad (\text{A.5})$$

and with respect to m_t :

$$- \delta_t + \beta E_{\{\theta_{t+1}, \varepsilon_{t+1}\}} (\gamma_{t+1} + \delta_{t+1}) \frac{1}{1 + \pi_{t+1}} = 0. \quad (\text{A.6})$$

From (A.3) and (A.4) we have that: $\frac{\mathcal{U}_{c,t}}{p_{c,t}} = \frac{\mathcal{U}_{u,t}}{p_{u,t}}$. Because of (A.2) equation (A.3) becomes $\frac{\mathcal{U}_{c,t}}{p_{c,t}} = \delta_t + \delta_t R_t$.

Also, from (A.6) and (A.3) we have: $\delta_t = \beta E_{\{\theta_{t+1}, \varepsilon_{t+1}\}} \frac{\mathcal{U}_{c,t+1}}{p_{c,t+1}} \frac{1}{1 + \pi_{t+1}}$. Using the last equation of the last paragraph we have that: $\frac{\mathcal{U}_{c,t}}{p_{c,t}} = E_{\{\theta_{t+1}, \varepsilon_{t+1}\}} \beta \frac{\mathcal{U}_{c,t+1}}{p_{c,t+1}} \frac{1}{1 + \pi_{t+1}} (1 + R_t)$.

Finally, from (A.5), (A.3) and (A.4) we have that $\frac{D'(H_t)}{w_t} = \frac{\mathcal{U}_{c,t}}{p_{c,t}} = \frac{\mathcal{U}_{u,t}}{p_{u,t}}$.

B Robustness Figures

Figure B.1: Percentage deviations from steady state after a one standard-deviation positive monetary policy shock for the constrained firms production, unconstrained firms production, total employment, employment of cash constrained firms, employment of unconstrained firms, relative employment of cash constrained firms, relative employment of unconstrained firms, cost of external financing and nominal interest rate. Note: $\phi_R = 0$ (Section 6.1).

Figure B.2: Percentage deviations from steady state after a one standard-deviation positive productivity shock for the constrained firms production, unconstrained firms production, total employment, employment of cash constrained firms, employment of unconstrained firms, relative employment of cash constrained firms, relative employment of unconstrained firms, cost of external financing and nominal interest rate. Note: $\phi_R = 0$ (Section 6.1).

Figure B.3: Percentage deviations from steady state after a one standard-deviation positive monetary policy shock for the constrained firms production, unconstrained firms production, total employment, employment of cash constrained firms, employment of unconstrained firms, relative employment of cash constrained firms, relative employment of unconstrained firms, cost of external financing and nominal interest rate. Note: $\kappa = 0.53$ (Section 6.1).

Figure B.4: Percentage deviations from steady state after a one standard-deviation positive productivity shock for the constrained firms production, unconstrained firms production, total employment, employment of cash constrained firms, employment of unconstrained firms, relative employment of cash constrained firms, relative employment of unconstrained firms, cost of external financing and nominal interest rate. Note: $\kappa = 0.53$ (Section 6.1).

Figure B.5: Percentage deviations from steady state after a one standard-deviation positive monetary policy shock for the constrained firms production, unconstrained firms production, total employment, employment of cash constrained firms, employment of unconstrained firms, relative employment of cash constrained firms, relative employment of unconstrained firms, cost of external financing and nominal interest rate. Note: $\xi = 5.6$ (Section 6.1).

Figure B.6: Percentage deviations from steady state after a one standard-deviation positive productivity shock for the constrained firms production, unconstrained firms production, total employment, employment of cash constrained firms, employment of unconstrained firms, relative employment of cash constrained firms, relative employment of unconstrained firms, cost of external financing and nominal interest rate. Note: $\xi = 5.6$ (Section 6.1).

Figure B.7: Percentage deviations from steady state after a one standard-deviation shock to α for the constrained firms production, unconstrained firms production, total employment, employment of cash constrained firms, employment of unconstrained firms, relative employment of cash constrained firms, relative employment of unconstrained firms, cost of external financing and nominal interest rate (Section 6.2). $\ln \alpha_t = (1 - \rho_\alpha) \ln \alpha + \rho_\alpha \ln \alpha_{t-1} + \epsilon_{\alpha,t}$, with $\epsilon_{\alpha,t} \sim \mathcal{N}(0, \sigma_\alpha^2)$. $\alpha = 1.155$, $\rho_\alpha = 0.9$ and $\sigma_\alpha = 0.03$.

Figure B.8: Percentage deviations from steady state after a one standard-deviation shock to κ for the constrained firms production, unconstrained firms production, total employment, employment of cash constrained firms, employment of unconstrained firms, relative employment of cash constrained firms, relative employment of unconstrained firms, cost of external financing and nominal interest rate (Section 6.2). $\ln \kappa_t = (1 - \rho_\kappa) \ln \kappa + \rho_\kappa \ln \kappa_{t-1} + \epsilon_{\kappa,t}$, with $\epsilon_{\kappa,t} \sim \mathcal{N}(0, \sigma_\kappa^2)$. $\kappa = 0.353$, $\rho_\kappa = 0.93$ and $\sigma_\kappa = 0.1$.