Inequity-averse preferences in general equilibrium

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Abstract

We study the stability with respect to the introduction of opportunity-based inequity aversion a la [Dufwenberg et al. \(2011\)](#) of three welfare properties satisfied by competitive equilibria in self-regarding economies: (i) Pareto efficiency may not be a stable property; (ii) undomination with respect to income redistribution is a stable property whenever the marginal indirect utility of income has no extreme variations; and (iii) generically (endowment-wise) market-constrained efficiency is a stable property.

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1 Introduction

We study welfare properties of competitive equilibria in economies with opportunity-based inequity-averse agents in the general equilibrium environment introduced by M. Dufwenberg, P. Heidhues, G. Kirchsteiger, F. Riedel, and J. Sobel (2011) —henceforth DHKRS.¹ Such an agent not only cares about her own

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¹Experimental economists have documented consistent human behavior that cannot be rationalized by self-regarding preferences. In some situations this behavior can be rationalized by inequity-averse preferences (see [Fehr and Schmidt, 2001](#), for a survey). DHKRS's opportunity-based preferences capture a form of inequity aversion that is plausible in a market, or any social system, in which agents are endowed with a set from which they can select an alternative to consume.

consumption, but also about the equitability of opportunities offered by the market to all agents. *Ceteris paribus*, her ideal is equality of opportunity. She loses welfare when her perception of the opportunities offered to others by the market deviate from those offered to her. The agent's preferences are parameterized, à la [Fehr and Schmidt \(1999\)](#), by the agent's internal utility function, which represents her subjective assessment of consumption bundles; a coefficient that captures the agent's aversion to inequity of opportunities against herself; and a coefficient that captures aversion to inequity of opportunities against others. When these two coefficients are zero, the agent exhibits no other-regarding behavior. As these coefficients grow, the agent's concern for the overall distribution of resources becomes more important compared to her private consumption. Remarkably, the set of competitive equilibria of an opportunity-based inequity-averse economy is exactly the set of competitive equilibria of the corresponding self-regarding economy (DHKRS).

Each equal-income competitive equilibrium in an opportunity-based inequity-averse economy is Pareto efficient (Proposition 1). Equal-income competitive equilibria were proposed by [Foley \(1967\)](#) in order to achieve no-envy, i.e., that no agent prefer the consumption of another agent to her own consumption. Proposition 1 adds to the inventory of normative considerations that points to these allocations as being central in the problem of equitably allocating resources (see [Thomson, 2010](#), for a survey).

Pareto efficiency of a competitive equilibrium with unequal incomes crucially depends on the absence of opportunity-based inequity aversion.² For any two-agent economy in which at least one agent's inequity aversion coefficients are positive, each competitive equilibrium with different incomes and whose outcomes are not ordered (with respect to the usual order in a Euclidean space) is not Pareto efficient. Moreover, given any two different convex internal preferences, one can find endowments such that the corresponding economy and all of its replicas possess a competitive equilibrium that is not Pareto efficient for economies with these internal preferences, endowments, and non-zero inequity aversion coefficients (Example 1).³

²It is well known that the First Welfare Theorem ([Arrow, 1951](#); [Debreu, 1951](#)) fails if there are externalities. Indeed, for almost all economies satisfying a separability condition, every competitive equilibrium is Pareto dominated by another competitive allocation for a suitably selected anonymous tax scheme ([Geanakoplos and Polemarchakis, 2008](#)). Opportunity-based inequity-aversion externalities do not belong in this domain.

³Indeed, the competitive equilibrium in Example 1 is Pareto dominated by a change in price without modifying consumption or endowment of the agents. Thus, even the weaker notion of efficiency that requires an allocation is not Pareto dominated by a change in price while keeping the initial endowments constant crucially depends on the self-regarding assumption for market outcomes.

Inefficiency of competitive equilibria in opportunity-based inequity-averse economies has a limit. First, for a neighborhood of each self-regarding economy whose marginal indirect utility of income has no extreme changes, a social planner who is constrained to make only income transfers at market prices, cannot improve upon any competitive equilibrium. More precisely, consider an economy such that for each agent the ratio of minimal to maximal marginal indirect internal utility of income for a given price is uniformly bounded across different prices. For each such an economy there is a neighborhood of zero in the inequity aversion parameter space for which the property holds for the corresponding other-regarding economies (Theorem 1) —homogeneous of degree one internal utility functions belong to this set (Corollary 2). This constrained form of efficiency is specially meaningful when there is a numeraire good that the social planner must use in order to reallocate income and whose consumption does not affect agents' market choice for the other goods (i.e., a quasi-linear environment). Second, generically (endowment-wise) there is a neighborhood of each smooth self-regarding economy in which no market outcome is Pareto dominated by another market outcome (Sec. 3.3). Thus, a social planner whose role is to select among possible market outcomes, generically cannot unequivocally improve upon any given market outcome in a neighborhood of each self-regarding economy.

Undomination by income redistribution is a property of the market that is more easily satisfied by large markets with opportunity-based inequity-averse agents (Theorems 2 and 3). This is because in Fehr and Schmidt's parameterization of inequity aversion, the effect of the consumption of a single agent in the utility of the other agents decreases with the size of the market. This makes it more difficult to achieve a Pareto improvement from a given allocation by redistributing income at market prices as the market grows. Indeed, for economies whose internal utility functions are selected from a finite set of homogeneous of degree one internal utility functions, there is no restriction on inequity aversion parameters in order to guarantee that as n is large each competitive allocation is undominated by income redistribution (Corollary 3).⁴

Our proof of Theorems 1-3, follow from a basic theorem that states conditions guaranteeing that in an opportunity-based inequity-averse economy a market outcome cannot be Pareto dominated by an allocation obtained by redistributing income at market prices (Theorem 4). First, the coefficient that captures the agent's aversion to inequity of opportunity against her, should sat-

⁴DHKRS first stated Corollary 3. However, they stated it as a consequence of a theorem that is mathematically incorrect (see last paragraph of the introduction of this paper for details). This is the first paper in which the statement is proved true.

isfy a parametric restriction that depends on the bounds for the income derivative of the agent's indirect internal utility function.⁵ This first restriction guarantees that the direct loss of at least one agent who gives up income in an income redistribution, cannot be compensated by the reduction in her perception that she is receiving more opportunities than the other agents. Second, for each agent, (i) either the indirect internal utility is a concave function of income, or (ii) her inequity aversion coefficients satisfy a joint restriction that depends on the bounds for the income derivative of the agent's indirect internal utility function. These additional requirements guarantee that for at least an agent who gives up income in an income redistribution, the part of her direct loss that cannot be compensated by the reduction in her perception that she is receiving more opportunities than the other, is not compensated by the reduction in her perception that other agents are getting better opportunities than her. If restriction (i) and (ii) above are violated, even by a minimal margin, Theorem 4 may not hold (Example 2).

Our Theorem 4 is closely related to Theorem 5.1 in DHKRS, which states conditions on preferences guaranteeing that a market outcome is undominated by income redistribution in an opportunity-based inequity-averse economy. A careful look at these authors' work reveals a subtle error. Essentially, they overlook the role of an agent's utility loss due to the agent's perception that other agents receive better opportunities.⁶ As a consequence they assert that there is no restriction on an agent's perception of inequity against herself in order to guarantee that a market allocation is undominated by income redistribution. This assertion is incorrect even if, *ceteris paribus*, one replicates agents in the economy (this is shown by Example 2). At a technical level, our work is closer to Velez (2015, Theorem 3), which states conditions guaranteeing an equal income competitive allocation is Pareto efficient in economies with indivisible goods.

⁵This was first observed by DHKRS.

⁶In Page 634, DHKRS claim the following in the paragraph after equation (A.5): one can take agent r , the one who loses the most income, to be, without loss of generality, the agent with highest income. This claim is based on the observation that proving equation (A.5) holds when one decreases the consumption of the agents who get income above agent r , actually proves equation (A.5). This is true. However, this does not imply the former claim. Equation (A.5) tells us about the utility of agent r , not the other agents. So when one makes the change, the agents who had higher income than agent r may lose utility. So an allocation that was better for them, necessarily becomes worse. Example 2 illustrates the issue: the agents who lose the most income are the medium-income agents.

2 Model

Consider the general equilibrium environment with opportunity-based other-regarding preferences introduced by DHKRS. There are L goods and prices are normalized so that $\mathbf{p} \geq 0$ and $\sum_{l=1}^L p_l = 1$. For simplicity in the presentation we will assume an exchange economy. The set of agents is $N \equiv \{1, \dots, n\}$. Each agent's consumption set is $\mathbb{R}_+^L$. Agent i 's **consumption bundle** is $\mathbf{x}_i \equiv (x_{i1}, \dots, x_{iL})$. The **consumption profile** is $\mathbf{x} \equiv (\mathbf{x}_i)_{i \in N}$. Agent i 's **endowment** is ω_i and the **profile of endowments** is $\omega \equiv (\omega_i)_{i \in N}$. An agent's welfare depends not only on her consumption, but also on the profile of budget sets in the society. That is, agents make a judgement about their opportunities and those of the other. We will concentrate on studying a market environment, so budget sets are determined by a vector of prices $\mathbf{p}$ and an **income profile**, $\mathbf{w} \equiv (w_i)_{i \in N}$. We will allow for income redistribution so we do not assume w_i is necessarily equal to $\mathbf{p} \cdot \omega_i$. Agent i 's **budget set** when her income is w_i and prices are $\mathbf{p}$ is $B_i(\mathbf{p}, w_i) \equiv \{\mathbf{x}_i \in \mathbb{R}^L : \mathbf{p} \cdot \mathbf{x}_i \leq w_i\}$; the **profile of budget sets** is denoted by $B(\mathbf{p}, \mathbf{w})$.

Feasibility of an allocation in an economy with opportunity-based inequity-averse agents goes beyond the simple aggregate availability of resources. The issue here is that agents are affected by the distribution of opportunities in the society. Thus in order for these perceived opportunities to be meaningful, one needs to require not only that aggregate consumption be equal to aggregate endowment, but also that each agent consume in her budget set (for possibly redistributed endowments).⁷ A **feasible allocation** is a triple $(\mathbf{x}, \mathbf{p}, \mathbf{w})$ such that $\sum_{i \in N} \mathbf{x}_i = \sum_{i \in N} \omega_i$, $\sum_{i \in N} w_i = \sum_{i \in N} \mathbf{p} \cdot \omega_i$, and for each i , $\mathbf{x}_i \in B(\mathbf{p}, w_i)$.

We consider agents who are averse to inequality of opportunities. That is, each agent not only cares about her own consumption, but also about the equitability of the profile of budget sets. The agent evaluates private consumptions by means of a utility function that we refer to as her **internal utility** and denote it by u_i . We denote the profile of internal utility functions by $\mathbf{u} \equiv (u_i)_{i \in N}$. We refer to the corresponding preferences as **internal preferences**. We assume that each u_i is continuous and represents locally non-satiated internal preferences, i.e., for each $\mathbf{x} \in \mathbb{R}_+^L$ and each $\varepsilon \in \mathbb{R}_{++}$, there is $\mathbf{y} \in \mathbb{R}_+^L$ such that $\|\mathbf{x} - \mathbf{y}\| < \varepsilon$ and $u_i(\mathbf{y}) > u_i(\mathbf{x})$.⁸ We denote the set of admissible internal utility functions by $\mathcal{U}$. Agent i 's evaluation of agent j 's budget set is given by agent i 's maximal internal utility in that set. In order to simplify notation, we denote the **indirect internal utility function** by $v^i(\mathbf{p}, w_i) \equiv \max_{\mathbf{x}_i \in B(\mathbf{p}, w_i)} u_i(\mathbf{x}_i)$. Agent i 's utility of

⁷Note that this notion of feasibility does not require that each agent's consumption maximizes her preferences in her budget set given what is assigned to the other. Requiring this would imply that only market allocations are feasible.

⁸Here $\|\cdot\|$ is the Euclidean distance.

allocation (x_i, p, w) , is given by:

$$V_i(u_i, c_i, a_i)(x_i, p, w) \equiv u^i(x_i) - \frac{a_i}{n-1} \sum_{j \in N} \max\{v^i(p, w_j) - v^i(p, w_i), 0\} \\ - \frac{c_i}{n-1} \sum_{j \in N} \max\{v^i(p, w_i) - v^i(p, w_j), 0\},$$

where a_i and c_i are non-negative constants such that $c_i < 1$ and $c_i \leq a_i$. Whenever it creates no unambiguity, we drop the parameters from $V_i(u_i, c_i, a_i)$ and simply write V_i . This domain of preferences generalizes a proposal by [Fehr and Schmidt \(1999\)](#) for the allocation of money when there are consumption externalities. We denote the set of admissible utility functions by $\mathcal{V}$ and the generic utility profile by $V \equiv (V_i)_{i \in N} \in \mathcal{V}^n$. We write $\mathcal{U}$ for the set of externality-free preferences. We denote the space of **utility-endowment pairs** by $\mathcal{E} \equiv \mathcal{V} \times \mathbb{R}_+^L$. The generic **economy** is $e \equiv (V, \omega) \in \mathcal{E}^n$.

We say that the **minimal ratio of marginal indirect internal utility of income is uniformly bounded for $u_i \in \mathcal{U}$** if for each price p the marginal indirect internal utility of income of u_i is between two positive constants $\underline{v}_l(p) < \overline{v}_l(p)$ and the ratio $\underline{v}_l(p)/\overline{v}_l(p)$ is bounded below by a positive number $c(u_i)$, which is independent of p . Let $\mathcal{B}$ be the domain of the internal utility functions satisfying this property. A familiar sub-domain of $\mathcal{B}$ is the one of internal utility functions that are homogeneous of degree one—for each such u_i , $c(u_i) = 1$.

A **competitive equilibrium** for (V, ω) is a pair (x, p) such that each agent maximizes in her budget set $B(p, p \cdot \omega_i)$ and market clears, i.e., $(x, p, (p \cdot \omega_i)_{i \in N})$ is feasible.⁹

Let $V \in \mathcal{V}^n$; (x, p, w) **Pareto dominates** (x', p', w') (for V) if for each agent (x, p, w) is at least as good as (x', p', w') and at least one agent prefers (x, p, w) to (x', p', w') . We consider four welfare properties that a market may have. Let (x, p) be a competitive equilibrium for $e \equiv (V, \omega)$; (x, p) is **efficient** (for e) if there is no feasible allocation (x', p', w') that Pareto dominates $(x, p, (p \cdot \omega_i)_{i \in N})$; (x, p) is **undominated by a price change** (for e), if there is no feasible allocation $(x', p', (p' \cdot \omega_i)_{i \in N})$ that Pareto dominates $(x, p, (p \cdot \omega_i)_{i \in N})$; (x, p) is **undominated by income redistribution at market prices** (for e), if there is no feasible allocation (x', p, w') that Pareto dominates $(x, p, (p \cdot \omega_i)_{i \in N})$; (x, p) is **market-constrained efficient** (for e), if there is no other competitive equilibrium (x', p') for e , such that $(x', p', (p' \cdot \omega_i)_{i \in N})$ Pareto dominates $(x, p, (p \cdot \omega_i)_{i \in N})$.

⁹A pair (x, p) is a competitive equilibrium of $((V_i(u_i, c_i, a_i))_{i \in N}, \omega)$ if and only if (x, p) is a competitive equilibrium of the no-externalities economy $((V_i(u_i, 0, 0))_{i \in N}, \omega)$ (see DHKRS). See [McKenzie \(2002\)](#) for a comprehensive treatment of existence of competitive equilibria in economies without externalities.

Let $\mathcal{D} \subseteq \mathcal{U}$. We say that a **property of the market is stable with respect to the introduction of opportunity-based inequity aversion in $\mathcal{D}$** if for each $e \equiv (u, \omega) \in (\mathcal{D} \times \mathbb{R}_+^L)^n$ there is $\varepsilon_e > 0$ such that whenever for each $i \in N$, $c_i < \varepsilon_e$ and $a_i < \varepsilon_e$, the property holds for each competitive equilibrium of $((V_i(u_i, c_i, a_i))_{i \in N}, \omega)$.

Let $\mathcal{D} \subseteq \mathcal{V}$. We say that the **market possesses a property for large n in $\mathcal{D}$** if there is $k \in \mathbb{N}$ such that for each $n \geq k$ and each $e \in (\mathcal{D} \times \mathbb{R}_+^L)^n$ we have that each competitive equilibrium of e satisfies the property.

Let $s \in \mathbb{N}$. The **s -replica of $e \equiv (V, \omega)$** is the economy for the set of agents $\{1, \dots, ns\}$ such that agents $\{1, \dots, s\}$ have preferences represented by V_1 and endowment ω_1 , agents $\{s+1, \dots, 2s\}$ have preferences represented by V_2 and endowment ω_2 , and so on. Let (x, p) be a competitive equilibrium for e . The **s -replica of (x, p)** is the competitive equilibrium for the s -replica of e , where agents $\{1, \dots, s\}$ consume x_1 , agents $\{s+1, \dots, 2s\}$ consume x_2 , and so on; and prices are p .

3 Results

3.1 Efficiency and undomination by a price change

Efficiency of equal-income competitive equilibria is not compromised by the introduction of opportunity-based inequity-aversion.

Proposition 1. *Each equal-income competitive equilibrium of an economy populated by opportunity-based inequity-averse agents is efficient.*

Proof. Let $e \equiv (V_i(u_i, c_i, a_i))_{i \in N}, \omega) \in \mathcal{E}^n$ and (x, p) a competitive equilibrium for e such that for each pair $\{i, j\} \subseteq N$, $p \cdot \omega_i = p \cdot \omega_j$. Let (x', p', w') be feasible. Suppose that there is $i \in N$ such that $V_i(x', p', w') > V_i(x, p, (p \cdot \omega_i))$. By definition of V_i , $u_i(x'_i) \geq V_i(x', p', w')$. Since (x, p) is an equal-income competitive equilibrium, then $V_i(x, p, (p \cdot \omega_i)) = u_i(x_i)$. Thus, $u_i(x'_i) > u_i(x_i)$. Since internal preferences are locally non-satiated, then (x, p) is efficient for the externality-free economy with preferences $(u_i)_{i \in N}$ and endowment profile ω . Thus, there is $j \in N$ such that $u_j(x_j) > u_j(x'_j)$. Thus, $V_j(x, p, w) = u_j(x_j) > u_j(x'_j) \geq V_j(x', p', w')$. Thus, (x, p, w) cannot be Pareto dominated by (x', p', w') and (x, p) is efficient for e . $\square$

Corollary 1. *Each competitive equilibrium of an equal-endowment economy populated by opportunity-based inequity-averse agents is efficient.*

Efficiency may be a fragile property of a competitive outcome in which agents have different income.

Example 1. Consider a self-regarding two-agent economy (V, ω) and a competitive equilibrium for it (x, p) . Suppose also that the equilibrium outcomes x_1 and x_2 are not ordered with respect to the Euclidean partial order and that $p \cdot x_1 < p \cdot x_2$. It is easy to see that there is a price vector $p' \geq 0$ such that $p' \cdot x_1 = p' \cdot x_2$. Thus, the allocation $(x_1, x_2, p', (p' \cdot x_1, p' \cdot x_2))$ is feasible and Pareto dominates $(x, p, (p \cdot \omega_1, p \cdot \omega_2))$ whenever $a_1 > 0$ or $c_2 > 0$. Moreover, the equilibrium (x, p) of economy (V, x) violates not only Pareto inefficiency, but also undomination by a price change (Figure 1). Note that these conclusions hold for arbitrarily replicas of the economy in the example.

Figure 1: Inefficient equilibrium with opportunity-based inequity-averse preferences.

Example 1 can be constructed with any two different strictly-monotone convex internal preferences as follows. Let u_1 and u_2 be two different externality-free strictly-increasing quasi-concave internal utility functions. Thus, there is a strictly positive price p and a strictly positive income level $w > 0$ such that the maximizers of u_1 and u_2 at prices p and income w can be selected to be different. Let x_1 and x_2 be these two maximizers. Since $p \cdot x_1 = p \cdot x_2$, there are $\{l, l'\} \in \{1, \dots, L\}$ such that $x_{2l} < x_{1l}$ and $x_{2l'} > x_{1l'}$. Let $\omega_1 = x_1$. Let $\varepsilon > 0$. By continuity and quas-concavity, one can find $\delta > 0$ such that a maximizer of u_2 in $B(p, w + \varepsilon)$, say x_2^ε , is ε close to its maximizer in $B(p, w)$ (c.f., [Mas-Colell et al., 1995](#)). In particular, one can select ε such that x_2^ε is not order related with x_1 . Let $\omega_2 \equiv x_2^\varepsilon$.

3.2 Undomination by income redistribution at market prices

If the marginal indirect internal utility of income for an agent can be either zero or arbitrarily large at some given prices, then a competitive equilibrium at these

prices may be dominated by income redistribution for any non-trivial inequity-averse preferences associated with the agent's internal utility (DHKRS). If one rules out these extreme conditions, undomination by income redistribution at market prices is a stable property of the market with respect to the introduction of opportunity-based inequity aversion.

Theorem 1. *Undomination by income redistribution at market prices is a stable property of the market with respect to the introduction of opportunity-based inequity aversion in $\mathcal{B}$.*

Corollary 2. *Undomination by income redistribution at market prices is a stable property of the market with respect to the introduction of opportunity-based inequity aversion in the domain of internal utility functions that are homogeneous of degree one.*

A joint restriction on inequity aversion parameters guarantees that the outcome of a large market is undominated by income redistribution.

Theorem 2. *Let $\mathcal{D} \equiv \{V_l\langle u_l, c_l, a_l \rangle\}_{l=1}^K \subseteq \mathcal{V}$ be a finite domain of utility functions. Suppose that for each $l = 1, \dots, K$, $u_l \in \mathcal{B}$ and $c_l + a_l(1 - c(u_l)) < c(u_l)$. Then, market is undominated by income redistribution at market prices for large n in $\mathcal{D} \cup \mathcal{U}$.*

If each agent's indirect internal utility is a concave function of income, there is no restriction of inequity-aversion against the agent herself in order to guarantee that for large n the market is undominated by income redistribution.

Theorem 3. *Let $\mathcal{D} \equiv \{V_l\langle u_l, c_l, a_l \rangle\}_{l=1}^K \subseteq \mathcal{V}$ be a finite domain of utility functions whose indirect internal utility, for each price, is a concave function of income. Suppose that for each $l = 1, \dots, K$, $u_l \in \mathcal{B}$ and $c_l < c(u_l)$. Then, market is undominated by income redistribution at market prices for large n in $\mathcal{D} \cup \mathcal{U}$.*

Corollary 3. *Let $\mathcal{D} \subseteq \mathcal{V}$ be a finite domain of utility functions whose internal utility is homogeneous of degree one. Then, market is undominated by income redistribution at market prices for large n in $\mathcal{D} \cup \mathcal{U}$.*

We now prove Theorems 1-3. We identify parametric restrictions on preferences that guarantee a competitive equilibrium is undominated by income redistribution at market prices. Essentially, one needs conditions that preclude the existence of an agent who by giving up some income to another agent, gains in utility because the direct effect of this redistribution is offset by the reduction in inequality induced by it. There are essentially two critical situations in which this can be so.

First, suppose that all agents $i \neq n$, have higher income than agent n . Suppose that each agent $i \neq n$ gives up one dollar to agent n . Each agent $i \neq n$ may gain in utility if the direct effect of the change is offset by the reduction in inequality. The following assumption guarantees this does not occur.¹⁰

Assumption F1 (at prices p): there are $\bar{v}_i > \underline{v}_i$ such that the income derivative of $v^i(p, \cdot)$ is bounded above by $\bar{v}_i$ and below by $\underline{v}_i$, and

$$\frac{c_i}{n-1} (\underline{v}_i + (n-1)\bar{v}_i) < \underline{v}_i.$$

The second critical situation in which an agent who gives up income may gain by an income redistribution is more subtle. Suppose that incomes are ordered $w_1 < w_2 < w_3 \leq \dots \leq w_n$. That is, agent 2 has the second lowest income. Suppose that agent 2 gives up $1 + \varepsilon$ in income, each agent $i > 2$ gives up 1 in income, and agent 1 receives the collected amount, $n - 1 + \varepsilon$. Evidently, agent 1 may benefit from the redistribution. The situation for agents 3, ..., n here is different than that described in the paragraph leading to assumption F1. Even if the indirect utility functions of these agents satisfy F1, they may gain by the change, because agent 2 is contributing a bit more than each of them to the transfer to agent 1. Now, if agent 2's marginal indirect utility around the income of agents 3, ..., 4, is higher than her marginal indirect utility of income around her income, she may be better off. This is so because the inequality against her is considerably reduced by the change. There are two ways in which one can guarantee this does not occur.¹¹

Assumption F2 (at prices p): there are $\bar{v}_i > \underline{v}_i$ such that the income derivative of $v^i(p, \cdot)$ is bounded above by $\bar{v}_i$ and below by $\underline{v}_i$, and

$$\frac{c_i}{n-1} (\underline{v}_i + (n-1)\bar{v}_i) + \frac{a_i}{n-1} (n-2)(\bar{v}_i - \underline{v}_i) \leq \underline{v}_i.$$

In contrast to the strict inequality in the definition of assumption F1, assumption F2 is defined by means of a weak inequality. This is so because this weak inequality is enough to guarantee that the agent is not willing to give up in income an amount $\varepsilon > 0$ more than what the agents with higher income give up.

¹⁰DHKRS first noticed that assumption F1 is a necessary condition to guarantee a competitive equilibrium for inequity-averse agents is never Pareto dominated by an income redistribution at market prices.

¹¹The role of aversion to inequity against the agent herself in the efficiency properties of competitive allocations for inequity-averse economies was first noticed by [Velez \(2015\)](#).

Observe that in the worst-case scenario prevented by *F2*, the agent who gives up the highest amount of income cannot be an agent with the highest income in the economy. Thus, this worst-case scenario can only happen when the marginal indirect utility of a low-income agent at a low income is lower than this agent's marginal indirect utility at higher income. The situation is precluded when marginal indirect internal utility is a decreasing function of income.

Assumption F3 (at prices p): Fixing p , the agent's indirect internal utility function is a concave function of income.

Theorems 1-3 are a direct consequence of the following result.

Theorem 4. Let (x, p) be a competitive equilibrium for (V, ω) . Assume that for each V_i one of the following is satisfied at prices p : (i) assumption *F1* and *F2*; (ii) assumption *F1* and *F3*; (iii) V_i is externality-free. Then (x, p) is undominated by income redistribution.

Proof. Let (V, ω) be such that V satisfies the assumptions of the theorem and (x, p) a competitive equilibrium for (V, ω) . We prove that (x, p) is efficient with respect to income redistribution. For each $i = 1, \dots, n$, let $w_i \equiv p \cdot \omega_i$. Since each agent is maximizing at the equilibrium in her own budget, it is enough to prove that if (x', p, w') is feasible and such that for each i , $V_i(x'_i, B(p, w')) \geq V_i(x_i, B(p, w))$, we have that for each $i = 1, \dots, n$, $w'_i \geq w_i$. Suppose by means of contradiction that there is j such that $w'_j < w_j$. Suppose without loss of generality that $j \in \arg \max\{w_i : k \in \arg \max_{i=1, \dots, n} w_k - w'_k\}$. That is, j is among the agents who lose income the most, one with the highest initial income. Denote $V_j(x_j, B(p, w)) - V_j(x'_j, B(p, w'))$ by $V_j - V'_j$. We prove that $V_j - V'_j > 0$. This contradicts our hypothesis.

Since we do not consider other indirect internal utility function than that of agent j , we drop the j subindex and superindex in the notation of v^j , $\underline{v}_j$, and $\bar{v}_j$. Since (x, p) is a competitive equilibrium for (V, ω) , then $V_j - V'_j > 0$ whenever V_j is externality-free. There are two remaining cases to prove that $V_j - V'_j > 0$.

Case 1: V_j satisfies *F1* and *F3*. For each $y \in \mathbb{R}^n$ and each $i = 1, \dots, n$, let

$$\Phi(y, i) \equiv -a_j \max\{v(y_i) - v(y_j), 0\} - c_j \max\{v(y_j) - v(y_i), 0\}.$$

Then,

$$V_j - V'_j = v(w_j) - v(w'_j) + \frac{1}{n-1} \sum_{i=1, \dots, n} \Phi(w, i) - \Phi(w', i).$$

For each $i = 1, \dots, n$, let $\Delta(i) \equiv \Phi(w, i) - \Phi(w', i)$. Let $A_1 \equiv \{i = 1, \dots, n : v(w_i) \leq v(w_j)\}$ and $i \in A_1$. We claim that $\Delta(i) \geq -c_j(v(w_j) - v(w'_j) + v(w'_i) - v(w_i))$. There are two cases.

Case 1: $v(w'_i) \leq v(w'_j)$. Then, $\Delta(i) = -c_j(v(w_j) - v(w_i)) + c_j(v(w'_j) - v(w'_i))$.

Case 2: $v(w'_i) > v(w'_j)$. Then, $\Delta(i) = -c_j(v(w_j) - v(w_i)) + a_j(v(w'_i) - v(w'_j)) \geq -c_j(v(w_j) - v(w_i))$. Now, $v(w_j) - v(w_i) = v(w_j) - v_j^i(w'_j) + v_j^i(w'_j) - v(w_i)$. By the case hypothesis, $v(w'_i) > v(w'_j)$. Thus, $v(w_j) - v(w_i) < v(w_j) - v_j^i(w'_j) + v_j^i(w'_i) - v(w_i)$. Thus, $\Delta(i) > -c_j(v(w_j) - v(w'_j) + v(w'_i) - v(w_i))$.

We claim that for each $i \in \{1, \dots, n\} \setminus A_1$, $\Delta(i) \geq 0$. Let $i \in \{1, \dots, n\} \setminus A_1$. Then, $v(w_i) > v(w_j)$ and $w_i > w_j$. By our choice of j as an agent with maximal income among the ones who lose the most income, we have that $w_j - w'_j > w_i - w'_i$. There are two cases.

Case 1: $w'_i \geq w_i$. Then, $v(w'_i) \geq v(w_i)$. Since $v(w_i) > v(w_j) > v(w'_j)$, then $v(w'_i) > v(w'_j)$. Thus, $\Delta(i) = -a_j(v(w_i) - v(w_j) + a_j(v(w'_i) - v(w'_j))) = a_j([v(w_j) - v(w'_j)] + [v(w'_i) - v(w_i)]) \geq 0$.

Case 2: $w'_i < w_i$. Recall that V_j satisfies F3. Thus, v is concave. Since $w_i > w_j$ and $w_j - w'_j \geq w_i - w'_i$, $v(w_i) - v(w'_i) \leq v(w_j) - v(w'_j)$. Thus, $v(w_i) - v(w_j) \leq v(w'_i) - v(w'_j)$. Since $v(w_j) < v(w_i)$, then $v(w'_j) < v(w'_i)$. Thus, $\Delta(i) = -a_j(v(w_i) - v(w_j)) + a_j(v(w'_i) - v(w'_j)) = a_j([v(w_j) - v(w'_j)] - [v(w_i) - v(w'_i)])$. Recall that $v(w_i) - v(w'_i) \leq v(w_j) - v(w'_j)$. Thus, $\Delta(i) \geq 0$.

Since for each $i \in A_1$, $\Delta(i) \geq -c_j(v(w_j) - v(w'_j) + v(w'_i) - v(w_i))$ and for each $i \in \{1, \dots, n\} \setminus A_1$, $\Delta(i) \geq 0$,

$$V_j - V'_j \geq v(w_j) - v(w'_j) - \frac{c_j}{n-1} \sum_{i \in A_1} v(w_j) - v(w'_j) + v(w'_i) - v(w_i).$$

If $c_j = 0$, then $V_j - V'_j > 0$. Thus, we can suppose without loss of generality that $\underline{v} > 0$, for otherwise F1 implies $c_j = 0$.

Suppose that $A_1 \neq \emptyset$, for otherwise our claim, $V_j - V'_j > 0$, would follow. Let $B \equiv \{i \in A_1 : w_i \leq w'_i\}$. Since the derivative of v is bounded above by $\bar{v}$ and bounded below by $\underline{v}$,

$$-\frac{c_j}{n-1} \sum_{i \in B} v(w'_i) - v(w_i) \geq -\frac{c_j}{n-1} \sum_{i \in B} \bar{v}(w'_i - w_i),$$

and

$$-\frac{c_j}{n-1} \sum_{i \in A_1 \setminus B} v(w'_i) - v(w_i) \geq -\frac{c_j}{n-1} \sum_{i \in A_1 \setminus B} \underline{v}(w'_i - w_i).$$

Thus,

$$-\frac{c_j}{n-1} \sum_{i \in A_1} v(w'_i) - v(w_i) \geq -\frac{c_j}{n-1} \left((\bar{v} - \underline{v}) \sum_{i \in B} (w'_i - w_i) + \underline{v} \sum_{i \in A_1} (w'_i - w_i) \right).$$

Recall that $\sum_{i \in A} w'_i \leq \sum_{i \in A} w_i$.¹² Thus, for any $C \subseteq \{1, \dots, n\}$,

$$\sum_{i \in C} w'_i - w_i \leq \sum_{i \in \{1, \dots, n\} \setminus C} w_i - w'_i \leq (n - |C|)(w_j - w'_j).$$

Denote by $1_{B \neq \emptyset}$ the indicator function that takes value 1 when $B \neq \emptyset$ and zero otherwise. Thus,

$$-\frac{c_j}{n-1} \sum_{i \in A_1} v(w'_i) - v(w_i) \geq -\frac{c_j}{n-1} (1_{B \neq \emptyset}(\bar{v} - \underline{v})(n - |B|) + \underline{v}(n - |A_1|))(w_j - w'_j).$$

Since the derivative of v is bounded below by $\underline{v}$, $\underline{v}(w_j - w'_j) \leq v(w_j) - v(w'_j)$. Thus,

$$-\frac{c_j}{n-1} \sum_{i \in A_1} v(w'_i) - v(w_i) \geq -\frac{c_j}{n-1} \left(\left(\frac{\bar{v} - \underline{v}}{\underline{v}} \right) (n-1) + (n - |A_1|) \right) (v(w_j) - v(w'_j)).$$

Thus, $V_j - V'_j > 0$ whenever

$$1 - \frac{c_j}{n-1} \left(|A_1| + \left(\frac{\bar{v} - \underline{v}}{\underline{v}} \right) (n-1) + (n - |A_1|) \right) > 0.$$

That is,

$$1 - \frac{c_j}{n-1} \left(\frac{(n-1)\bar{v} + \underline{v}}{\underline{v}} \right) > 0.$$

By F1, $c_j < \frac{(n-1)\underline{v}}{\bar{v} + (n-1)\underline{v}}$. Thus, $V_j - V'_j > 0$.

Case 2: V_j satisfies F1 and F2. Theorem 4 clearly follows if $c_j = a_j = 0$. Thus, we can suppose without loss of generality that $\underline{v} > 0$, for otherwise F2 implies $c_j = a_j = 0$. Note that the only place that F3 was used above was to prove that for each $i \in \{1, \dots, n\} \setminus A_1$ such that $w'_i < w_i$, $\Delta(i) \geq 0$. Now that F3 may not be satisfied, we prove instead that for each $i \in \{1, \dots, n\} \setminus A_1$ such that $w'_i < w_i$, $\Delta(i) > -a_j \left(\frac{\bar{v}}{\underline{v}} - 1 \right) (v(w_j) - v(w'_j))$. Let $i \in \{1, \dots, n\} \setminus A_1$ be such an agent. Since $i \in \{1, \dots, n\} \setminus A_1$,

$$\begin{aligned} \Delta(i) &= -a_j(v(w_i) - v(w_j)) + c_j \max\{v(w'_j) - v(w'_i), 0\} \\ &\quad + a_j \max\{v(w'_i) - v(w'_j), 0\}. \end{aligned}$$

¹²Observe that our argument applies when there is free disposal of money, for a modified bound in which the set D at the end of our estimation can be taken to be such that $|D| = n - 1$.

Since $w_i > w'_i$, $i \notin A_1$, and j is an agent with maximal income among the ones who lose the most income, $w_j - w'_j > w_i - w'_i$. Thus, we have that $v(w_i) - v(w'_i) \leq \bar{v}(w_i - w'_i) < \bar{v}(w_j - w'_j) \leq \frac{\bar{v}}{\underline{v}}(v(w_j) - v(w'_j))$. Then,

$$v(w_i) - v(w'_i) < \frac{\bar{v}}{\underline{v}}(v(w_j) - v(w'_j)). \quad (1)$$

There are two cases.

Case 1: $v(w'_i) < v(w'_j)$. Then,

$$\Delta(i) = -a_j(v(w_i) - v(w_j)) + c_j(v(w'_j) - v(w'_i)).$$

By our hypothesis, $v(w'_j) > v(w'_i)$. Thus, $\Delta(i) \geq -a_j(v(w_i) - v(w_j))$. Now, $v(w_i) - v(w'_i) = v(w_i) - v(w_j) + v(w_j) - v(w'_j) + v(w'_j) - v(w'_i)$. By the case hypothesis, $v(w'_j) > v(w'_i)$. Thus,

$$v(w_i) - v(w_j) \leq v(w_i) - v(w'_i) - (v(w_j) - v(w'_j)).$$

Our claim then follows from equation (1).

Case 2: $v(w'_i) \geq v(w'_j)$. Then,

$$\Delta(i) = -a_j(v(w_i) - v(w_j)) + a_j(v(w'_i) - v(w'_j)).$$

Thus, $\Delta(i) = -a_j(v(w_i) - v(w'_i)) + a_j(v(w_j) - v(w'_j))$. Our claim then follows from equation (1).

Let $D \equiv \{i \in \{1, \dots, n\} \setminus A_1 : w'_i < w_i\}$. Recall that $j \in A_1$. Thus, the cardinality of D is at most $n - 2$, for otherwise $\sum_{i \in A} w'_i < \sum_{i \in A} w_i$. If $D = \emptyset$, then our argument when $F1$ and $F3$ hold applies. Suppose then that $|D| > 0$. Since the remaining of the argument when $F1$ and $F3$ is valid, we have that

$$\begin{aligned} V_j - V'_j &> (v(w_j) - v(w'_j)) \left(1 - \frac{c_j}{n-1}(|A_1| + \frac{\bar{v}}{\underline{v}}|A \setminus A_1|) - \frac{a_j}{n-1} \left(\frac{\bar{v}}{\underline{v}} - 1 \right) |D| \right) \\ &\geq (v(w_j) - v(w'_j)) \left(1 - c_j \left(\frac{v+(n-1)\bar{v}}{(n-1)\underline{v}} \right) - a_j \frac{n-2}{n-1} \left(\frac{\bar{v}}{\underline{v}} - 1 \right) \right), \end{aligned}$$

where the strict inequality follows from $|D| > 0$. By $F2$, $V_j - V'_j > 0$. $\square$

If the assumptions in Theorem 4 are not satisfied, the result may not hold. We construct an economy and a competitive equilibrium for it that is Pareto dominated by an income redistribution at market prices. Each V_i in our example satisfies $F1$, but violates $F2$ and $F3$. The example is constructed along the lines of the worst-case scenario described before we introduce assumption $F2$. The main difference between our example and the situation described there is that we consider an economy in which $n - 2$ agents have the second lowest income. We do so in order to exhibit an economy that preserves all of its features as one replicates it an arbitrary number of times.

Example 2. To simplify notation we construct an example in which there are common bounds on marginal indirect utility of income for all agents. These common bounds are $\underline{\nu} = 1$ and $\overline{\nu} = K > 1$. Since K can be arbitrarily close to 1, one can say that our example shows that Theorem 4 may not hold even for arbitrarily small violations of F3.¹³ Our economy has sr agents with $r \geq 3$ such that $1/(r-2) < K-1$ and $s \geq 1$; our economy is the s -replica of an r -agent economy. We choose preferences for which the derivative of the indirect internal utility functions are bounded below by 1 and above by K . We select coefficients c_i satisfying F1, i.e., such that for each $i \in N$,

$$c_i < \frac{sr-1}{1+(sr-1)K}. \quad (2)$$

The limit of the expression above as $s \rightarrow \infty$ is $1/K$. Thus, if one selects $c_i < 1/K$, there is $S_1 \in \mathbb{N}$ such that for each $s \geq S_1$ equation (2) is satisfied.

Let $p \gg 0$ be a positive price. For simplicity we denote agent i 's income, i.e., $p \cdot \omega_i$, by w_i . Since in what follows prices are constant, we drop them from the notation. We choose internal preferences that are homothetic and such that at prices p all agents maximize on the ray with direction $\Omega \equiv \sum_{i \in N} \omega_i$.¹⁴ This guarantees that each balanced income redistribution is feasible. More precisely, with our assumption, agent i 's equilibrium consumption, which we denote by x_i , is proportional to the aggregate endowment. This proportion is exactly the proportion of her income in the aggregate income. Let α_i be such that $x_i = \alpha_i \Omega$. Since $p \cdot x_i = w_i$, we have that $\alpha_i = \frac{1}{p \cdot \Omega} w_i$. In the same way, if the agent is reassigned an income w'_i , her optimal consumption in $B_i(p, w'_i)$ is $x'_i = (1/p \cdot \Omega) w'_i \Omega$. Suppose that the income redistribution $w' \equiv (w'_i)_{i \in N}$ is balanced, i.e., $\sum_{i \in N} w_i = \sum_{i \in N} w'_i$. This implies that

$$\sum_{i \in N} x'_i = \sum_{i \in N} \left(\frac{1}{p \cdot \Omega} w'_i \right) \Omega = \frac{1}{p \cdot \Omega} \sum_{i \in N} w'_i \Omega = \Omega.$$

Thus, (p, x', w') is feasible.

Notice that the homothetic requirement of internal preferences is an ordinal requirement. We choose a particular representation for these preferences

¹³If one modifies our example by setting $s = 1$ and making the set of medium-income agents the singleton $\{2\}$, one can show that even if F2 is violated for a minimal margin, Theorem 4 may not hold.

¹⁴For concreteness one can take $L = 2$, symmetric Cobb-Douglas internal preferences, i.e., preferences represented by $(x_{i1}, x_{i2}) \mapsto x_{i1} x_{i2}$, $p = (1, 1)$, and Ω in the diagonal of $\mathbb{R}^2$. This Ω is specified later when each agent's income is defined. We remark that even though one can take Cobb-Douglas preferences for this example, internal utility cannot be taken to be one of its homogeneous representations. Instead, internal utility must be recalibrated as specified later in the example.

that exhibits some further properties. We calibrate this representation by assigning values of utility along the ray of direction Ω . We do this with an increasing function that will be defined later for each agent. To be more precise, let $\hat{u}_i : \mathbb{R}_+ \rightarrow \mathbb{R}$ be an increasing function. Define $u_i : \mathbb{R}_+^L \rightarrow \mathbb{R}$ as follows. For each $x_i \in \mathbb{R}_+^L$, $u_i(x_i) \equiv \hat{u}_i(\alpha_i)$ where $\alpha_i \geq 0$ is the unique factor such that the agent's internal preferences exhibit indifference between $\frac{\alpha_i}{p \cdot \Omega} \Omega$ and x_i . Clearly, u_i represents the agent's internal preferences.

Notice that by homotheticity of preferences agent i 's indirect utility function for prices p is $\hat{u}_i$. More precisely, $v^i(p, w'_i) = u_i((w'_i/p \cdot \Omega)\Omega) = \hat{u}_i(w'_i)$. Thus, we can complete our example by defining values for $v^i(p, \cdot) = \hat{u}_i(\cdot)$ that are consistent with monotonicity with respect to income and the bounds on marginal indirect utility of income. We can do so because representation of internal homothetic preferences imposes no restriction on $\hat{u}_i$ besides monotonicity.

In our basic r -agent economy, which will be replicated s times, agents are partitioned in three groups: low-income agent $L \equiv \{1\}$; medium-income agents $M \equiv \{2, \dots, r-1\}$; and high-income agent $H \equiv \{r\}$. Agent 1 exhibits no externalities, i.e., $c_1 = a_1 = 0$ and has initial income $w_1 = 0$. Thus, this agent is made better off by any positive transfer of income to her.

Step 1: *If each medium-income agent gives up $1+\varepsilon$ in income and each high-income agent gives up 1 in income, no medium-income agent is worse off.*

Let $1/(r-2) < \varepsilon < K-1$. Agents in M all have identical income and preferences. We then specify data for a generic medium-income agent m . Agents in H also have equal income and preferences. We then specify data for a generic high-income agent h . Incomes are ordered, $w_h > w_m > w_1 = 0$. Moreover, $w_h - 1 > w_m$ and $w_m - (1 + \varepsilon) > 1 + (r-2)(1 + \varepsilon)$. This assumptions guarantee that after each of the high-income agents gives up 1 in income, each medium-income agent gives up $(1 + \varepsilon)$ in income, and agent 1 receives the collected amount, $1 + (r-2)(1 + \varepsilon)$, the order of incomes in the society does not change.

Let $c_m = 0$. In order to guarantee that a medium-income agent is willing to give up $1 + \varepsilon$, we need that:

$$v^m(w_m - (1 + \varepsilon)) - \frac{a_m}{sr-1} s(v^m(w_h - 1) - v^m(w_m - (1 + \varepsilon))) \geq$$

$$v^m(w_m) - \frac{a_m}{sr-1} s(v^m(w_h) - v^m(w_m)).$$

That is,

$$s \frac{a_m}{sr-1} (v^m(w_h) - v^m(w_h - 1)) \geq \left(1 + s \frac{a_m}{sr-1}\right) (v^m(w_m) - v^m(w_m - (1 + \varepsilon))).$$

In order to maintain our derivatives condition, suppose that $v^m(w_m) - v^m(w_m - (1 + \varepsilon)) = 1 + \varepsilon$ and $v^m(w_h) - v^m(w_h - 1) = K$. One can select v^m , i.e., $\hat{u}^m$, that is increasing, smooth, and satisfies these two conditions (Figure 3). Since $\varepsilon < K - 1$, the inequality above is equivalent to:

$$a_m \geq \frac{(sr - 1)(1 + \varepsilon)}{s(K - (1 + \varepsilon))}.$$

Thus, we select a_m such that:

$$a_m \geq \frac{r(1 + \varepsilon)}{K - (1 + \varepsilon)}. \quad (3)$$

The following is happening here. Medium-income agents have less income than high-income agents. Changes in income in the region where high-income agents consume are more important for medium-income agents than changes in the region where they consume. Thus, if each medium-income agent cares enough about the inequity against her, she will pay $1 + \varepsilon$ in order for high-income agents to give up 1. Note that (3), our lower bound on the parameter a_m , is independent of s . Thus, after K is fixed, r is chosen so $1/(r - 2) < K$, and a_m is chosen to satisfy (3), medium agents are not worse off by the income redistribution in any s -replica of the r -agent economy.

Step 2: *High-income agents are willing to give up 1 in exchange for medium-income agents giving up $1 + \varepsilon$ and transferring the collected amount, $1 + (r - 2)(1 + \varepsilon)$, to agent 1.*

Again, consider the income reallocation at prices p at which each high-income agent gives up 1 in income, each medium-income agent gives up $1 + \varepsilon$ in income, and low income agents receive each $1 + (r - 2)(1 + \varepsilon)$. Obviously, agent 1 prefers the change. Moreover, no medium-income agent is worse off (Step 1). It remains to prove that we can find c_h satisfying equation (2) such that each high-income agent prefers the change. Notice that we have not made any assumption about the internal utility of high-income agents, so we have complete freedom to make choices, of course, bounded by our derivative assumptions. Indeed, suppose that $v^h(w_h) - v^h(w_h - 1) = 1$, $v^h(w_m) - v^h(w_m - (1 + \varepsilon)) = 1 + \varepsilon$, and $v^h(1 + (r - 2)(1 + \varepsilon)) - v^h(0) = K(1 + (r - 2)(1 + \varepsilon))$. That is, high-income agents are more sensitive to changes in income in the region where the income of agent 1 changes than in the region where the income of all other agents change. One can select v^h , i.e., $\hat{u}^h$, that is increasing, smooth, and satisfies these three conditions (Figure 3). Recall also that high-income agents remain the agents with highest income after the income redistribution. Thus,

Figure 2: Indirect internal utility functions for medium-income and high-income agents.

no high-income agent is worse off with the income redistribution whenever:

$$\begin{aligned} & v^h(w_h - 1) - \frac{c_h}{sr-1} s(r-2)(v^h(w_h - 1) - v^h(w_m - (1+\varepsilon))) \\ & - \frac{c_h}{sr-1} s(v^h(w_h - 1) - v^h(1 + (r-2)(1+\varepsilon))) \geq \\ & v^h(w_h) - \frac{c_h}{sr-1} s(r-2)(v^h(w_h) - v^h(w_m)) - \frac{c_h}{sr-1} s(v^h(w_h) - v^h(0)). \end{aligned}$$

That is,

$$\begin{aligned} & s \frac{c_h}{sr-1} (v^h(1 + (r-2)(1+\varepsilon)) - v^h(0) + v^h(w_h) - v^h(w_h - 1)) \geq \\ & (1 - s(r-2) \frac{c_h}{sr-1}) (v^h(w_h) - v^h(w_h - 1)) \\ & + s(r-2) \frac{c_h}{sr-1} (v^h(w_m) - v^h(w_m - (1+\varepsilon))). \end{aligned}$$

By our assumptions on v^h , this is:

$$s \frac{c_h}{sr-1} (K(1 + (r-2)(1+\varepsilon)) + 1) \geq (1 - s(r-2) \frac{c_h}{sr-1}) + s(r-2) \frac{c_h}{sr-1} (1 + \varepsilon).$$

Equivalently,

$$c_h \geq \frac{sr-1}{s} \frac{1}{K(1 + (r-2)(1+\varepsilon)) + 1 - (r-2)\varepsilon}. \quad (4)$$

Now, the limit as $s \rightarrow \infty$ of the expression above on the right is:

$$\frac{r}{K(1+(r-2)(1+\varepsilon))+1-(r-2)\varepsilon}.$$

Thus, we can find c_h and $S_2 \in \mathbb{N}$ such that for each $s \geq S_2$, c_h satisfies equations (2) and (4) whenever,

$$\frac{r}{K(1+(r-2)(1+\varepsilon))+1-(r-2)\varepsilon} < \frac{1}{K}. \quad (5)$$

This inequality holds whenever $\varepsilon < 1/(r-2)$.

Summarizing: fix $K > 1$, r such that $1/(r-2) < K-1$, and ε such that $1/(r-2) < \varepsilon < K-1$, the value functions for low-income agent, medium-income agents, and high-income agent defined above, $c_m = 0$, a_m satisfying (3), and c_h in the interior of the interval defined by the two expressions in (5). There is S such that for each $s \geq S$, the preferences of all agents satisfy *FI* and there is a competitive equilibrium that is dominated by income redistribution at market prices. $\square$

3.3 Market-constrained efficiency

A competitive equilibrium in an opportunity-based inequity-averse economy may be Pareto dominated by another competitive equilibrium of the same economy. This is easily seen in an economy in which one competitive equilibrium has unequal incomes and another has equal-incomes. If inequity aversion parameters are large enough for the agents whose internal utility is higher in the unequal-income equilibrium, this equilibrium is Pareto dominated by the equal-income equilibrium. If preferences are smooth, generically (endowment-wise) these situations do not arise when inequity-aversion parameters are small. This can be seen as follows. First, market-constrained efficiency of competitive equilibrium in an economy that has single-valued demands and finitely many competitive equilibria is a property that is stable with respect to the introduction of opportunity-based inequity aversion (Proposition 2 below). Second, fix $u \equiv (u_i)_{i \in N} \in \mathcal{U}^n$ satisfying the following two conditions: (i) each agent has a single-valued continuously differentiable demand function; and (ii) for at least one agent, the Euclidean norm of her demand converges to infinity for each sequence of price-wealth vectors that converge to a boundary price and a positive wealth. Then, the space of endowment vectors $\omega \in \mathbb{R}_{++}^L$ for which (u, ω) has infinitely many competitive equilibria has null closure with respect to the Lebesgue measure (Debreu, 1970).

Proposition 2. *Let $\mathcal{C} \subseteq (\mathcal{U} \times \mathbb{R}_+^L)^n$ be the domain of externality-free economies with single-valued demands and finitely many competitive equilibria. Then market-constrained efficiency is a stable property of the market with respect to opportunity-based inequity aversion in $\mathcal{C}$.*

Proof. Let $e \equiv ((u_i)_{i \in N}, \omega) \in \mathcal{C}$ and (x, p) and (x', p') be two different competitive equilibria for e . Since demands are single valued, then $p \neq p'$. Since x and x' are Pareto efficient for e , there must be $i \in N$ such that $u_i(x'_i) > u_i(x_i)$. As $(c_i, a_i) \rightarrow (0, 0)$, $V_i\langle u_i, c_i, a_i \rangle(x, p, (p \cdot \omega_j)_{j \in N}) \rightarrow u_i(x_i)$. Thus, there is $\varepsilon > 0$ such that if $c_i < \varepsilon$ and $a_i < \varepsilon$,

$$V_i\langle u_i, c_i, a_i \rangle(x', p', (p' \cdot \omega_j)_{j \in N}) > V_i\langle u_i, c_i, a_i \rangle(x, p, (p \cdot \omega_j)_{j \in N}).$$

Since there are finitely many equilibria, one can find $\varepsilon > 0$ such that if $c_i < \varepsilon$ and $a_i < \varepsilon$, no competitive equilibrium of e , is Pareto dominated within the set of competitive equilibria of e for $(V_i\langle u_i, c_i, a_i \rangle)_{i \in N}$. $\square$

Any restriction on individual preferences that guarantees uniqueness of competitive equilibria for all possible endowments, guarantees stability of market-constrained efficiency of competitive equilibria—these conditions are well understood and very restrictive (see [Mas-Colell, 1989](#)). It is difficult to guarantee stability of market-constrained efficiency in economies in which endowments are unrestricted, i.e., by only imposing restrictions on preferences or internal utility. The situation is no better even if one is willing to consider finitely many preferences and evaluate market-constrained efficiency of large economies generated by the finite set. In particular, the parallel statement to Corollary 3 that instead of undomination with respect to income redistribution at market prices involves market-constrained efficiency fails for each subdomain of homothetic preferences. That is, given a “regular” homothetic preference, one of its continuous representations u_i , and any admissible positive c_i and a_i , one is able to construct a two-agent economy $(V_i\langle u_i, c_i, a_i \rangle, u_j, \omega)$ with a competitive equilibrium that is not market-constrained efficient for any replica of the economy.

Example 3. Let $N \equiv \{1, 2\}$, u^1 be a continuous but otherwise arbitrary internal utility that represents a strictly increasing, strictly convex, smooth, and homothetic preference in $\mathbb{R}_+^2$, and $a_1 > 0$. We construct u^2 and $\omega \equiv (\omega_i)_{i \in N}$ such that there is a competitive equilibrium that is inefficient for the two-agent economy (V, ω) where $c_2 = a_2 = 0$.

Figure 3: Inefficient equilibrium with opportunity-based inequity-averse preferences.

Let $\Omega \gg 0$ and p the support of the upper contour set of u^1 at Ω . Since u^1 is strictly increasing, then $p \gg 0$. Let $p' \equiv (p_1, p'_2) \gg 0$ be such that $0 < p'_2 < p_2$ and $h' \equiv \arg \min_{x \in \mathbb{R}_+^2: u^1(x) \geq u^1(\Omega)} p' \cdot x$. Since the preference represented by u^1 is strictly convex, $h' \neq \Omega$. Since a Hicksian demand satisfies the uncompensated law of demand, $(p' - p) \cdot (h' - \Omega) < 0$ (Mas-Colell et al., 1995). Thus, $(p'_2 - p_2)(\Omega_2 - h'_2) < 0$ and $h'_2 > \Omega_2$. Thus, $h'_1 < \Omega_1$, for otherwise $p' \cdot h' > p' \cdot \Omega$. Let $p'' \equiv (p_1, p''_2) \gg 0$ such that $0 < p''_2 < p'_2$ and $h'' \equiv \arg \min_{x \in \mathbb{R}_+^2: u^1(x) \geq u^1(\Omega)} p'' \cdot x$. By the same argument, $h''_2 > h'_2 > \Omega_2$ and $h''_1 < h'_1 < \Omega_1$. The horizontal intercept of the hyperplane with normal p'' that passes through h'' is $a \equiv (p'' \cdot h'' / p_1, 0)$. We prove that $p' \cdot h' > p' \cdot a$. By definition of h'' , $p'' \cdot h'' < p'' \cdot h'$. Thus,

$$p'' \cdot h'' - p' \cdot h' < p'' \cdot h' - p' \cdot h' = (p'' - p') \cdot h' = (p''_2 - p'_2)h'_2 < 0.$$

Since $p' \cdot a = p'' \cdot a = p'' \cdot h''$, we have that $p' \cdot a < p' \cdot h'$.

Now, since $p' \cdot a < p' \cdot h'$ and u^1 represents a continuous preference, there is ω_2 such that $p'' \cdot \omega_2 = p'' \cdot h''$ and $p' \cdot \omega_2 < p' \cdot h'$. Thus,

$$v^1(p', p' \cdot \omega_2) < v^1(p', p' \cdot h') = v^1(p'', p'' \cdot \omega_2). \quad (6)$$

Let $\alpha \in (0, 1)$, $\omega_1 \equiv \alpha\Omega$, y be the ray with direction h'' and such that $p'' \cdot \omega_1 = p'' \cdot y$, and z on the ray with direction h' and such that $p' \cdot \omega_1 = p' \cdot z$. Since $y_2/y_1 > \omega_{12}/\omega_{11}$, $y_2 > \omega_{12}$, for otherwise $y_1 < \omega_{11}$ and $p'' \cdot y < p'' \cdot \omega_1$. The same argument shows that $z_2 > \omega_{12}$. Since $p'' \cdot y = p'' \cdot \omega_1$, we have that $p' \cdot y + (p'_2 - p''_2)y_2 = p' \cdot \omega_1 + (p'_2 - p''_2)\omega_{12}$. Thus, $p' \cdot y - p' \cdot \omega_1 = (p'_2 - p''_2)(y_2 - \omega_{12}) > 0$, and $p' \cdot y > p' \cdot \omega_1 = p' \cdot z$. Since $p' \cdot z = p' \cdot \omega_1$, then $p'' \cdot z + (p'_2 - p''_2)z_2 = p'' \cdot \omega_1 + (p'_2 - p''_2)\omega_{12}$. Thus, $p'' \cdot z - p'' \cdot \omega_1 = (p'_2 - p''_2)(\omega_{12} - z_2) < 0$ and $p'' \cdot z < p'' \cdot \omega_1 = p'' \cdot y$.

Let $x = (x_i)_{i \in N}$ be given by $x_1 = y$ and $x_2 = \omega_2 + (\omega_1 - y)$ and $x' = (x'_i)_{i \in N}$ be given by $x'_1 = z$ and $x'_2 = \omega_2 + (\omega_1 - z)$. By selecting α close enough to 0 one can guarantee $x \gg 0$ and $x' \gg 0$. Let u^2 be an internal utility whose maximizer in $B(p'', p'' \cdot \omega_2)$ is x_2 and whose maximizer in $B(p', p' \cdot \omega_2)$ is x'_2 (for instance u_2 's indifference set at ω_2 may be linear with normal p'' and the upper contour set of u_2 at x'_2 is in the intersection of the two half spaces $p'' \cdot y > p'' \cdot \omega_2$ and $p' \cdot y \geq p' \cdot \omega_2$). By construction, $(x, p'', (p'' \cdot \omega_i)_{i \in N})$ is a competitive equilibrium for V . We claim that for α small enough $(x', p', (p' \cdot \omega_i)_{i \in N})$ Pareto dominates $(x, p'', (p'' \cdot \omega_i)_{i \in N})$. Since $p'' \cdot z < p'' \cdot y$, $p'' \cdot x < p'' \cdot x'$. Thus, for each $s \geq 1$, $V_2(s * x_2, p'', (p'' \cdot s * \omega_i)_{i \in N}) < V_2(s * x'_2, p', (p' \cdot s * \omega_i)_{i \in N})$, where $s * x$, $s * x'$, and $s * \omega$ denote the s -replica of the consumption vectors and endowment. Now, for α small enough so agent 1 has the smallest income at both prices p'' and p' , $V_1(s * x_1, p'', (p'' \cdot s * \omega_i)_{i \in N}) < V_1(s * x'_1, p', (p' \cdot s * \omega_i)_{i \in N})$, if and only if

$$u^1(y_1) - \frac{a_1}{2s-1} s(v^1(p'', p'' \cdot \omega_2) - u^1(y_1)) < u^1(z_1) - \frac{a_1}{2s-1} s(v^1(p', p' \cdot \omega_2) - u^1(z_1)).$$

Equivalently,

$$\left(\frac{2}{a_1} - \frac{1}{s a_1} + 1 \right) (u^1(y_1) - u^1(z_1)) < v^1(p'', p'' \cdot \omega_2) - v^1(p', p' \cdot \omega_2).$$

By (6) the right hand side of the inequality above is positive. Since u^1 is continuous and both y and z converge to 0 as α converges to 0, then there is α such that the inequality above holds for all $s \geq 1$. $\square$

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